Nonlinear Analysis of Reinforced Concrete Structures in Design and Structural Assessment

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Outline:

Červenka Consulting - Computer simulation (virtual testing) of concrete structures

Finite element system ATENA – theoretical background, structure, practical applications

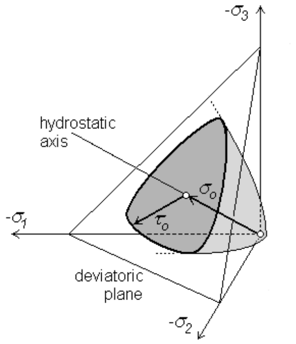
Contents

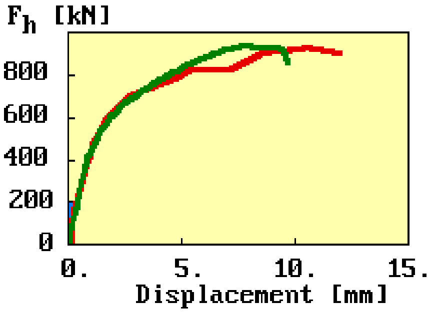
1. **What is simulation?**

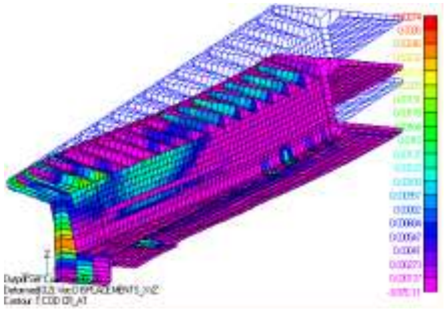
1. **Numerical models for the simulation of reinforced concrete:**
    1. Nonlinear finite element analysis
    2. Material models, fracture-plastic, microplane
    3. Special FE for reinforced concrete modeling

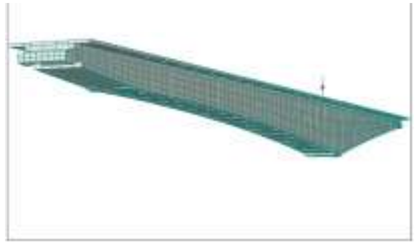
- **Validation:**
    - Tension stiffening
    - Round robin predictions
    - Full scale structural tests

- **Applications:**
    - Bridges
    - Tunnels
    - Nuclear containment





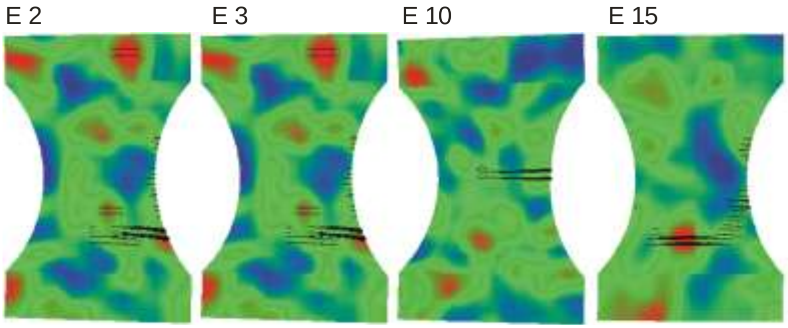


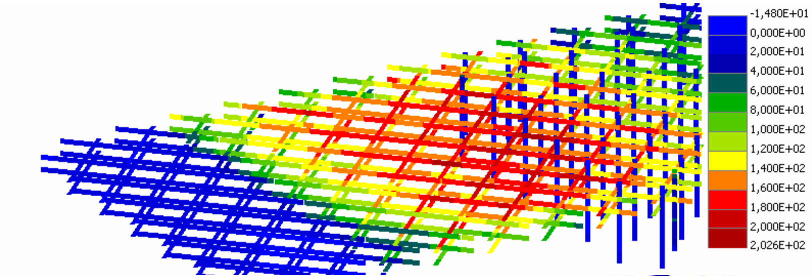


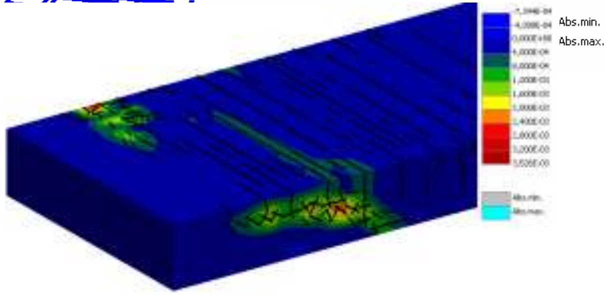
Why nonlinear simulation of structures?

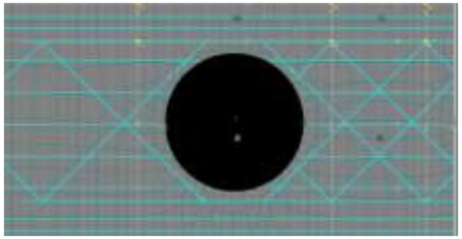
Supports expert engineering knowledge

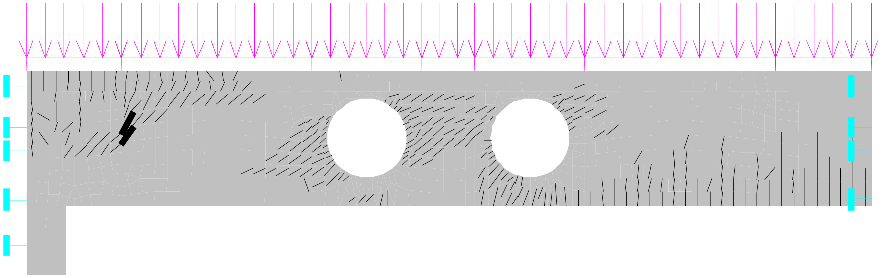
Nonlinear simulation of reinforced concrete structures









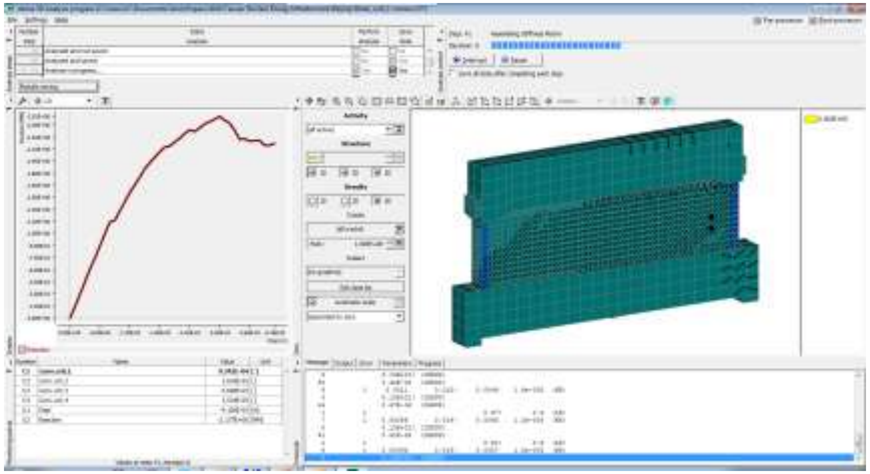


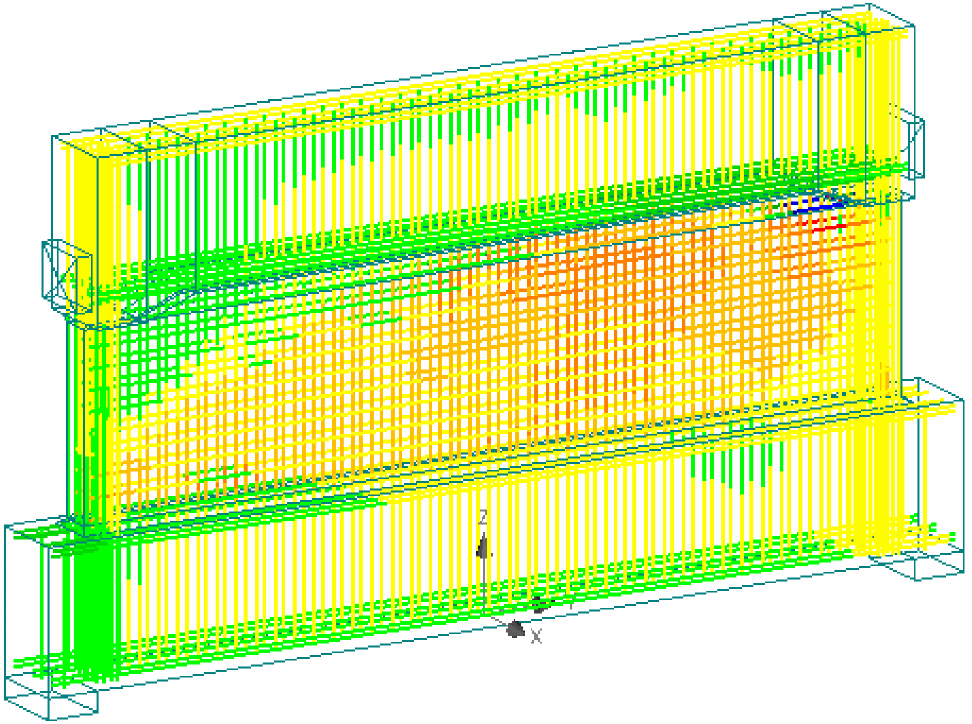
ATENA:

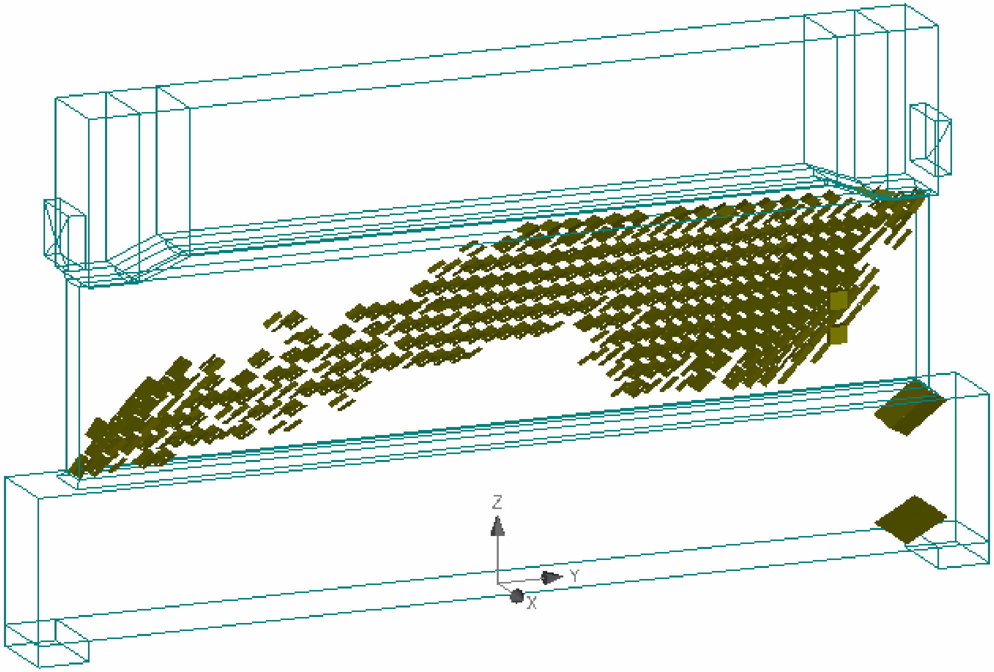
reinforcement modeling

realistic crack display

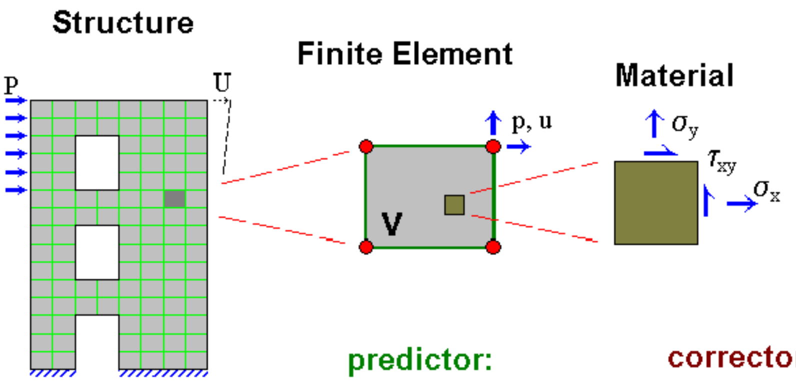
run-time visualization

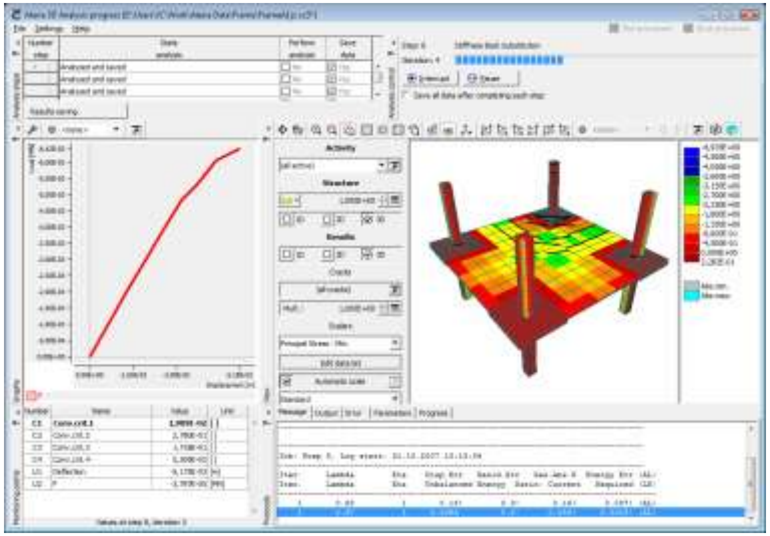




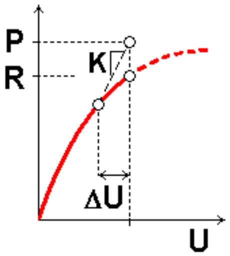


Nonlinear Finite Element Analysis





Non-linear Solution:



predictor:

$$\varepsilon = B u$$

$$\sigma = D \varepsilon$$

$$k = \int_v B^T D B \, dv$$

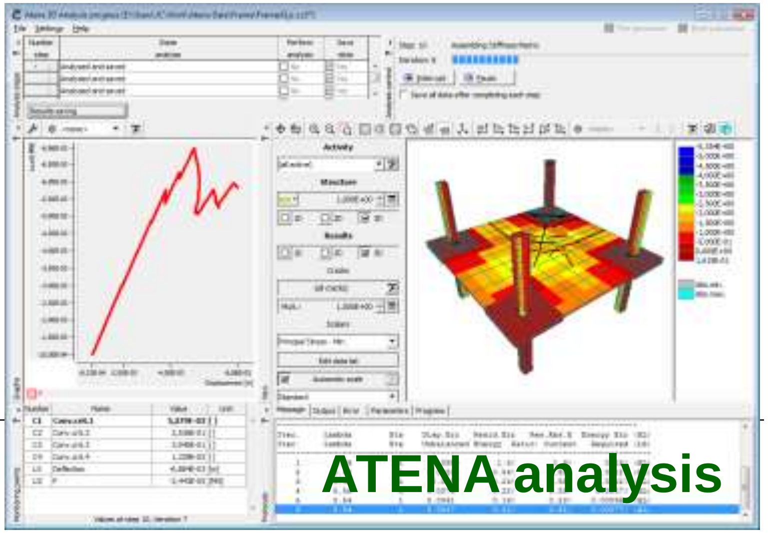
corrector:

$$\sigma = F_{(\sigma,\varepsilon)}$$

$$r = \int_v B^T \sigma \, dv$$

Equilibrium:

$$K \Delta U = P - R$$



ATENA analysis

Nonlinear constitutive models in ATENA

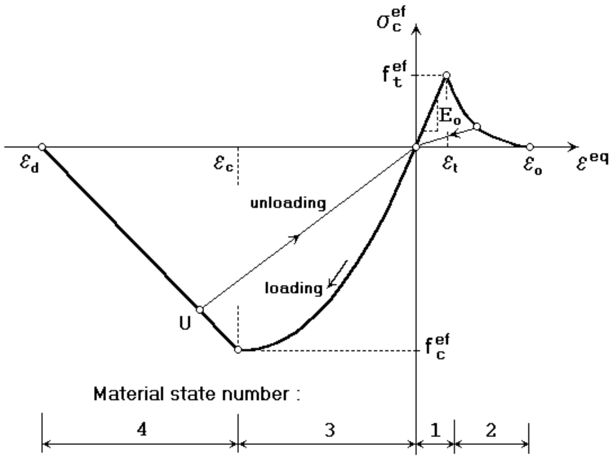
variety of nonlinear material models:

- for concrete
  - plain
  - reinforced
  - pre-stressed
  - fibre reinforced
- other quasi-brittle materials
  - masonry
  - rock
  - soil
  - metals

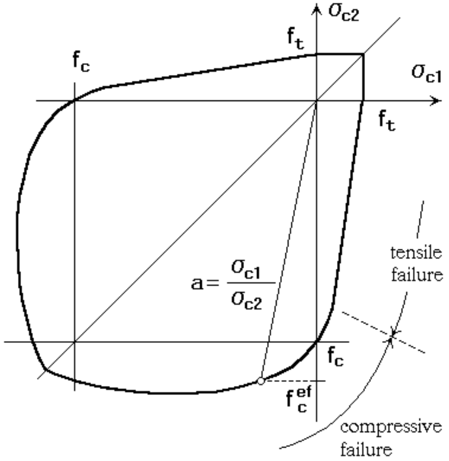
Material Models for Concrete

Plasticity
Damage mechanics
Microplane models

Uniaxial law

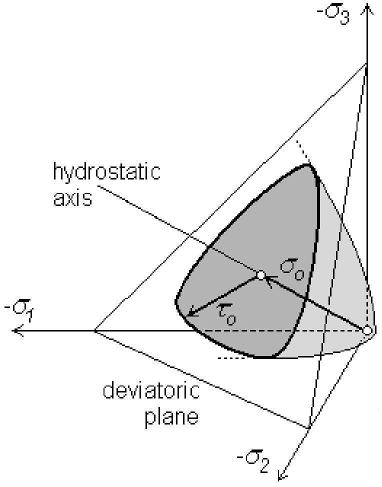


Bi-axial criterion



Kupfer 1969

3D failure surface



Menetrey Willam, ACI 1995

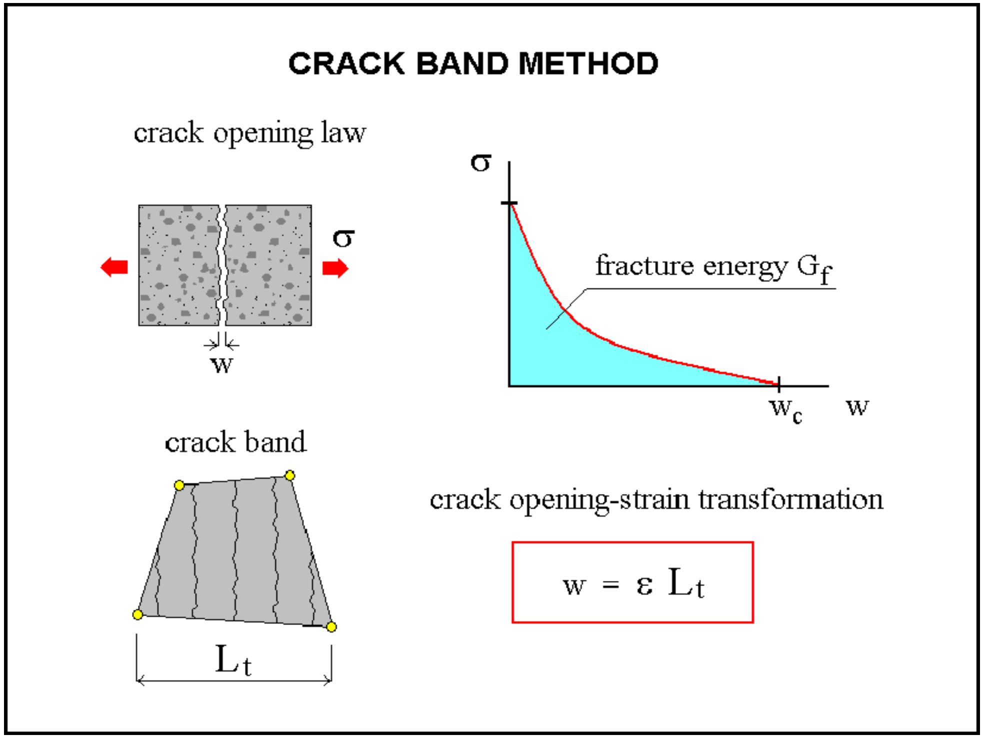
Program ATENA

Crack band method – correct energy dissipation during the fracturing process

concrete in tension

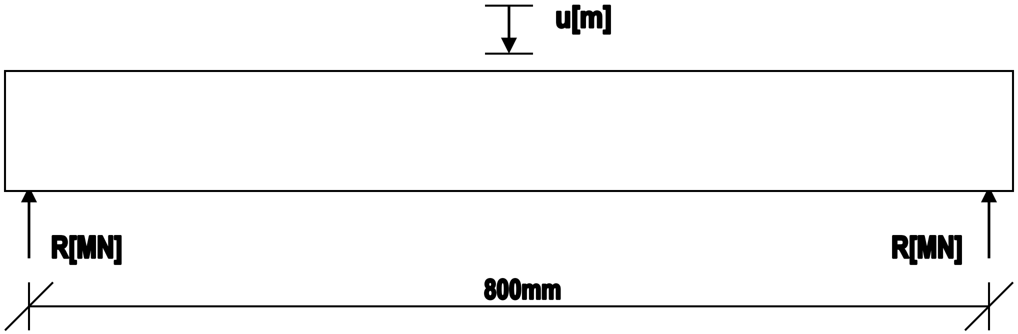
- tensile cracks
- post-peak behavior
- fracture energy

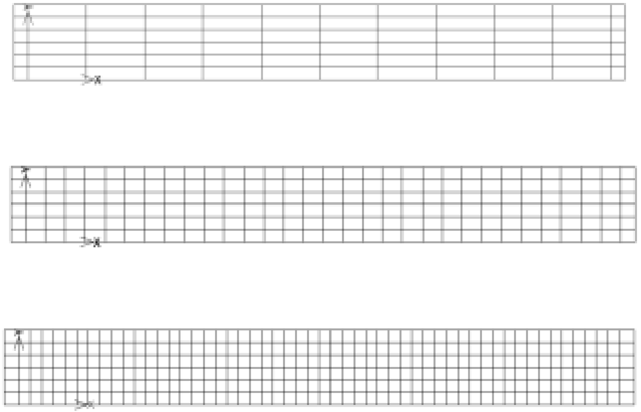
- crack band method

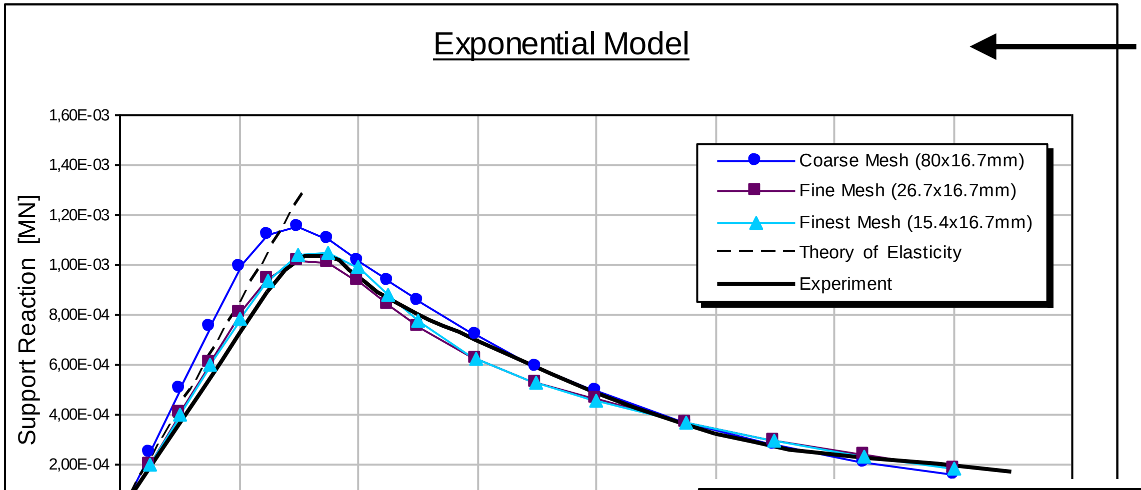


Demonstration Examples – mesh objectivity

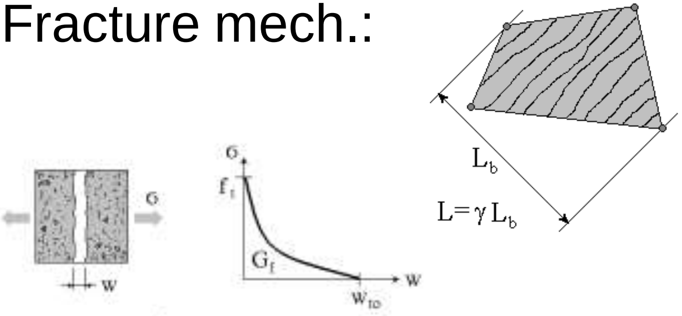
Importance of Fracture mechanics x Stress-strain laws







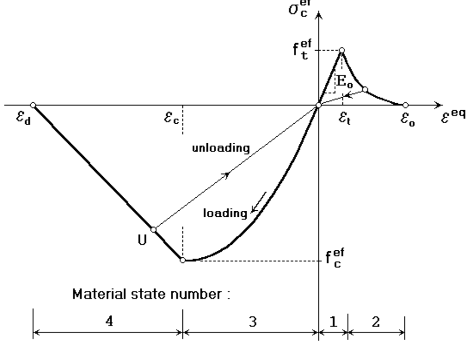
Fracture mech.:

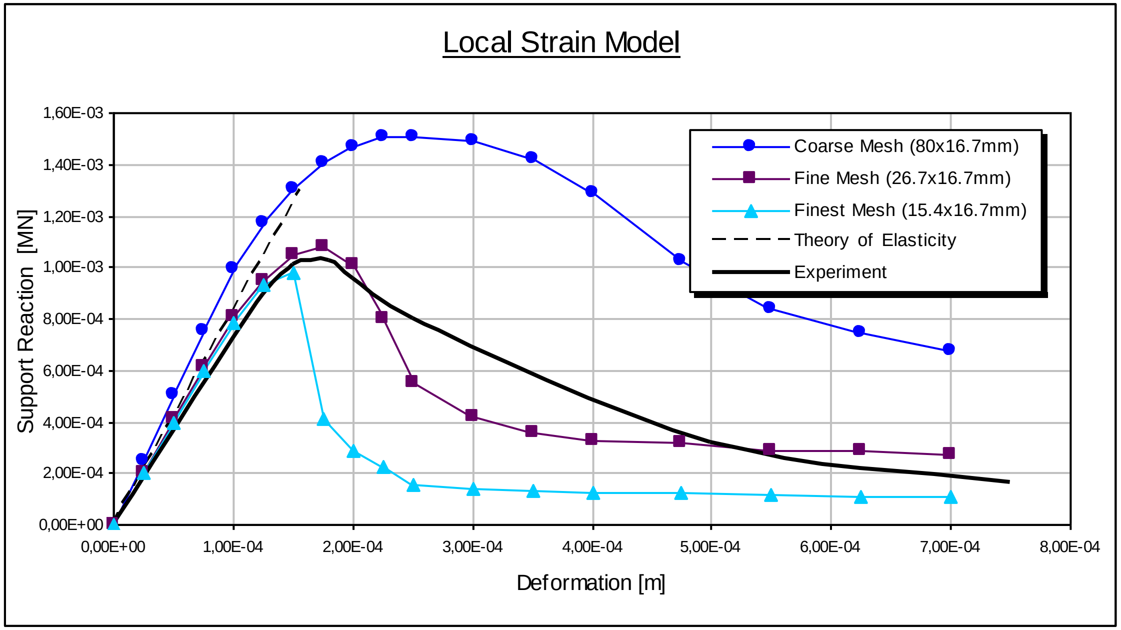


$$\frac{\sigma}{f_t}=\left(1+c_1\left(\frac{w}{w_{tc}}\right)^3\right)\exp\left(-c_2\frac{w}{w_{tc}}\right)-\frac{w}{w_{tc}}\left(1+c_1^3\right)\exp\left(-c_2\right)$$

$c_1 = 3,\quad c_2 = 6.93,\quad w_{tc} = 5.14\,\frac{G_f}{f_t}$

Local model:





Program ATENA

Numerical core - nonlinear material models

concrete in tension

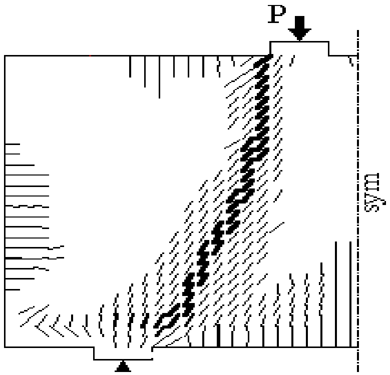
tensile cracks
post-peak behavior

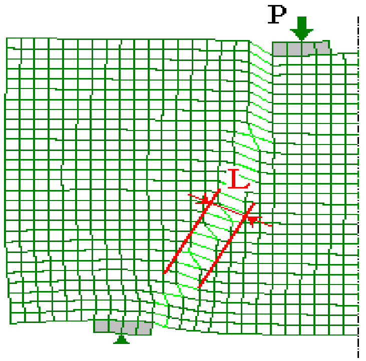
crack band method
fracture energy

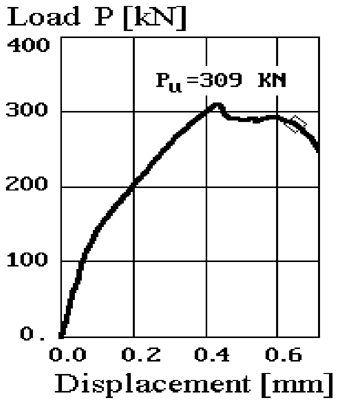
fixed or rotated cracks
crack localization
deterministic size-effect
is captured

Crack band size: L

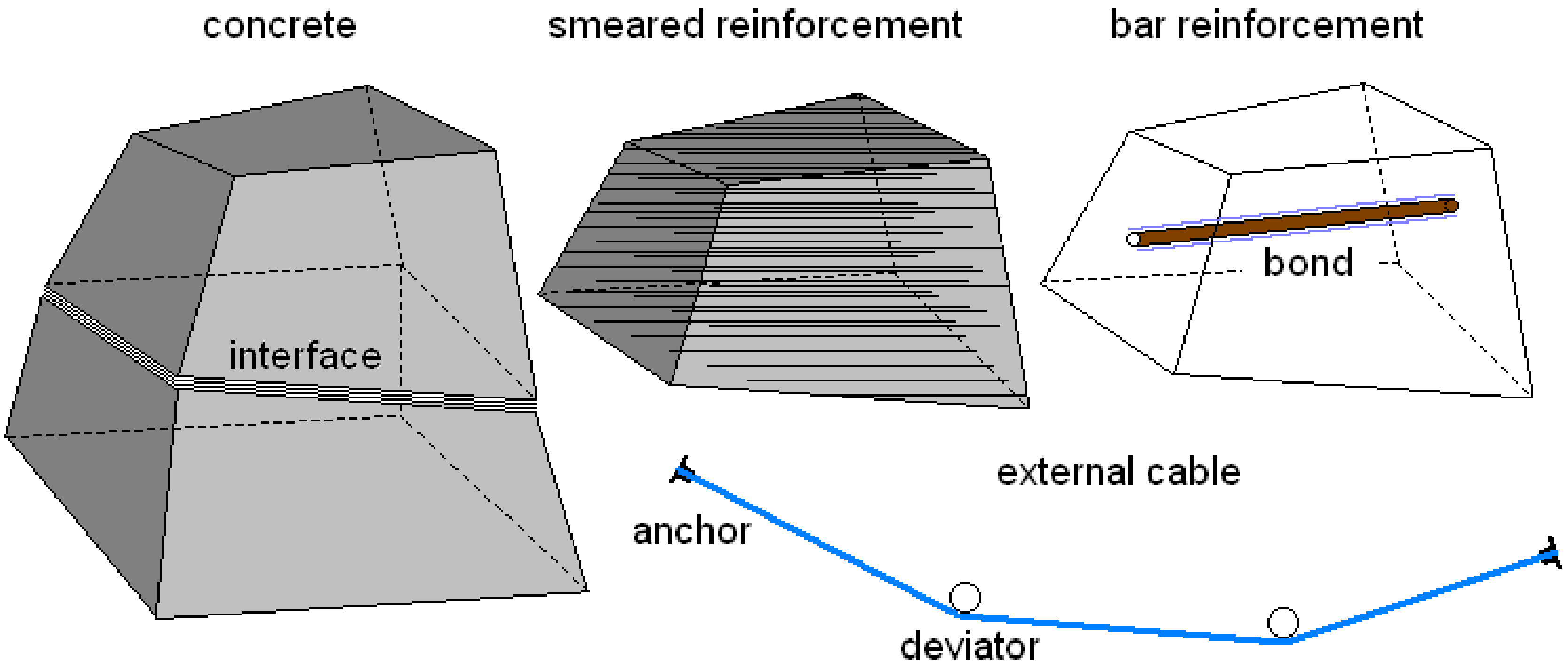
$$e = \frac{w}{L}$$



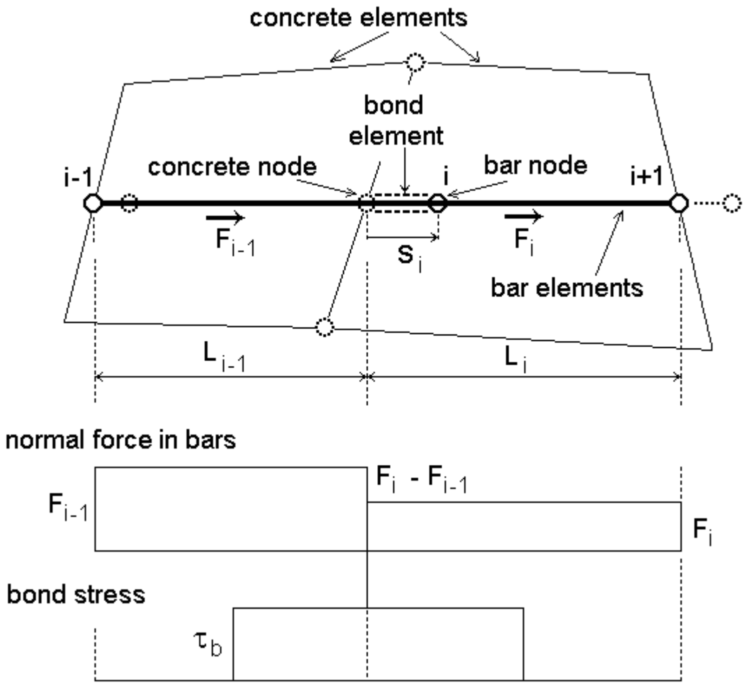


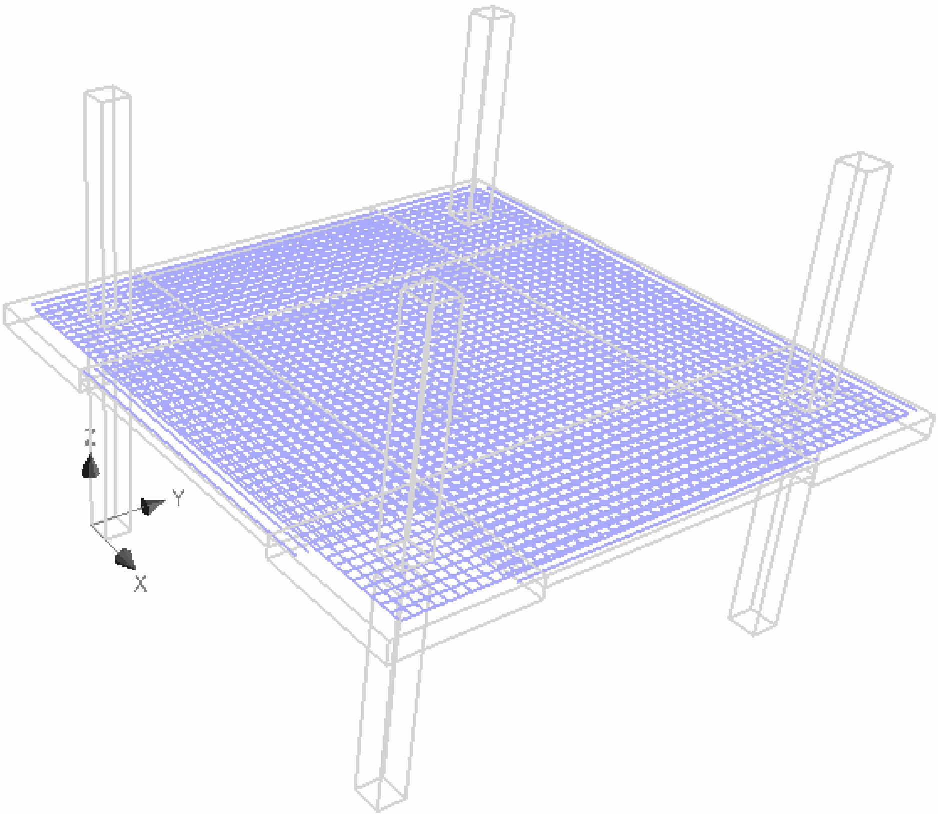


Special elements for reinforced concrete analysis



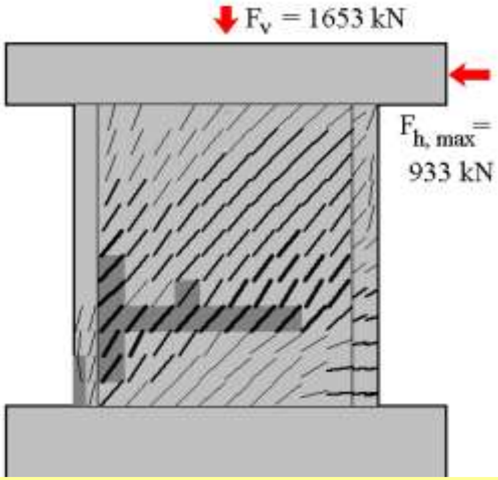
Reinforcement bond model



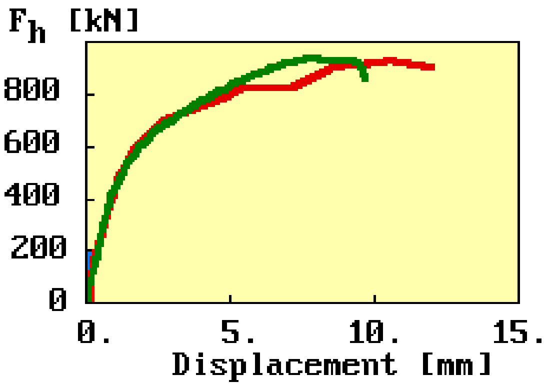


VALIDATION: Simulation of laboratory experiments

Reality



Simulation



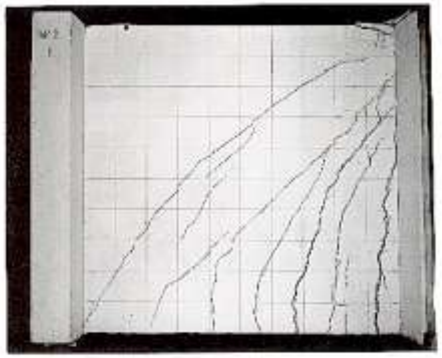
Validation and Reliability
Blind Predictions

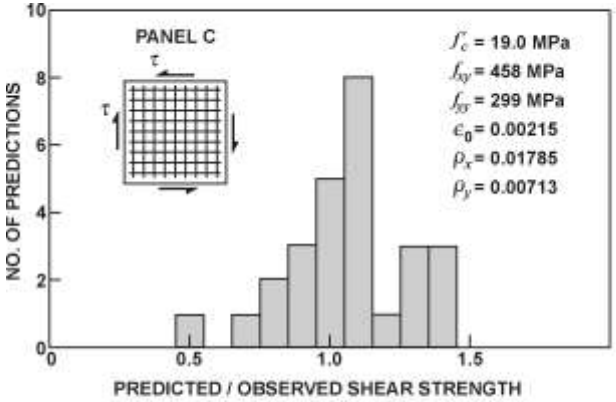
Toronto Panel (Collins, Melhorn) 1986 competition results (Panel C).

a) Variations in predicted shear strength

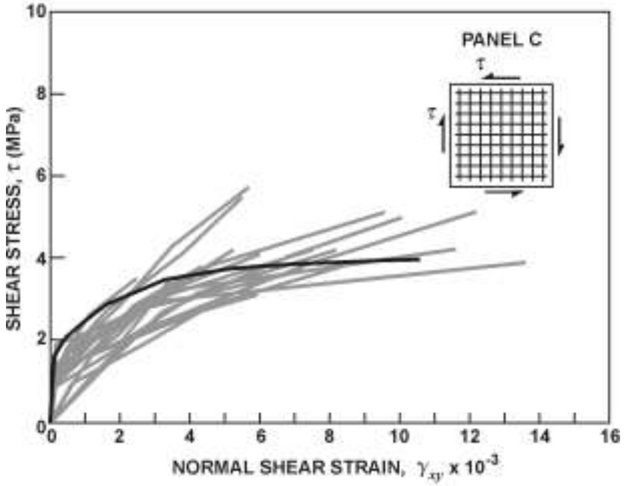
b) Variations in predicted load-deformation response

winner Vladimir Cervenka



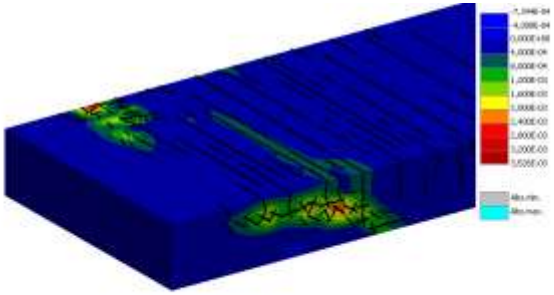


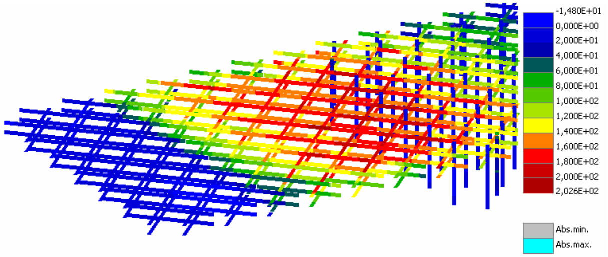
(a)



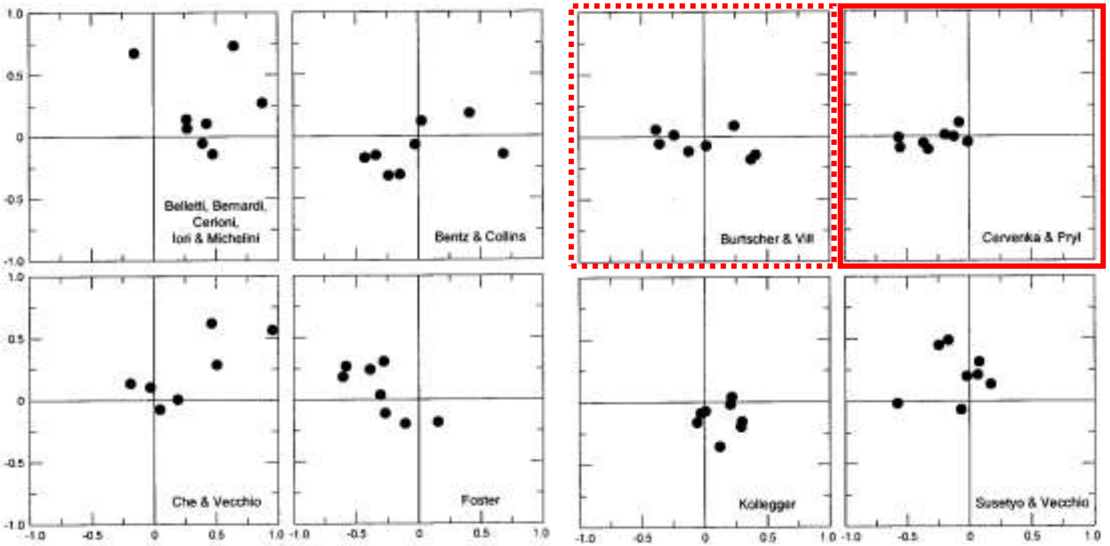
(b)

Validation: Round Robin Competition, Marti 2005





ATENA predictions

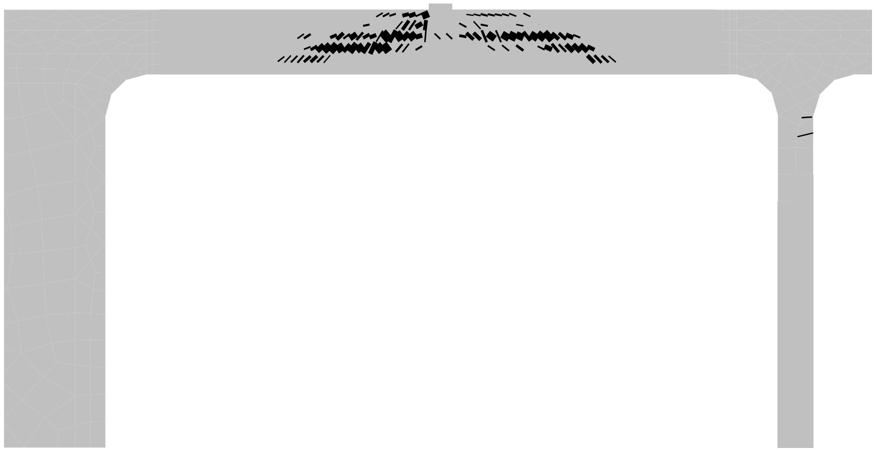


Validation: Field Test – Örnsköldsvik, Sweden

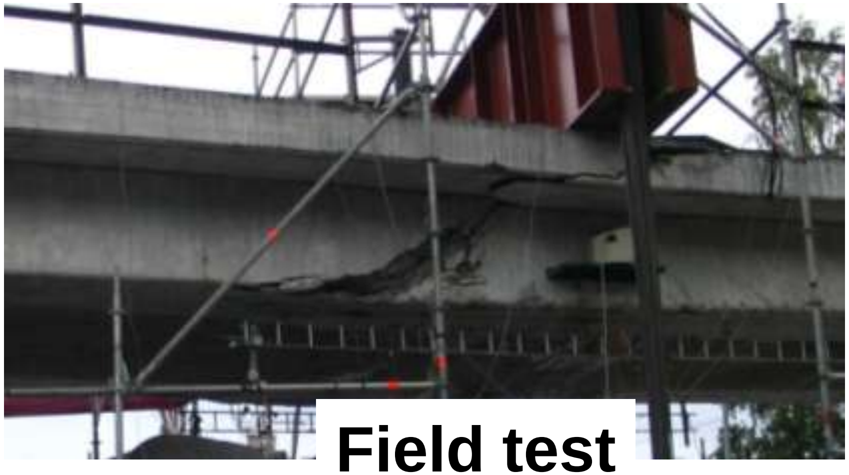
Sustainable Bridges

Validation: Field Test
Örnsköldsvik, Sweden
Final failure

Step 40,
Cracks: in elements, <5.000E-03; ...), openning: <-4.092E-04;4.226E-02>[m], Sigma_n: <-1.912E+01;2.009E+00>[MPa], Sigma_T : <



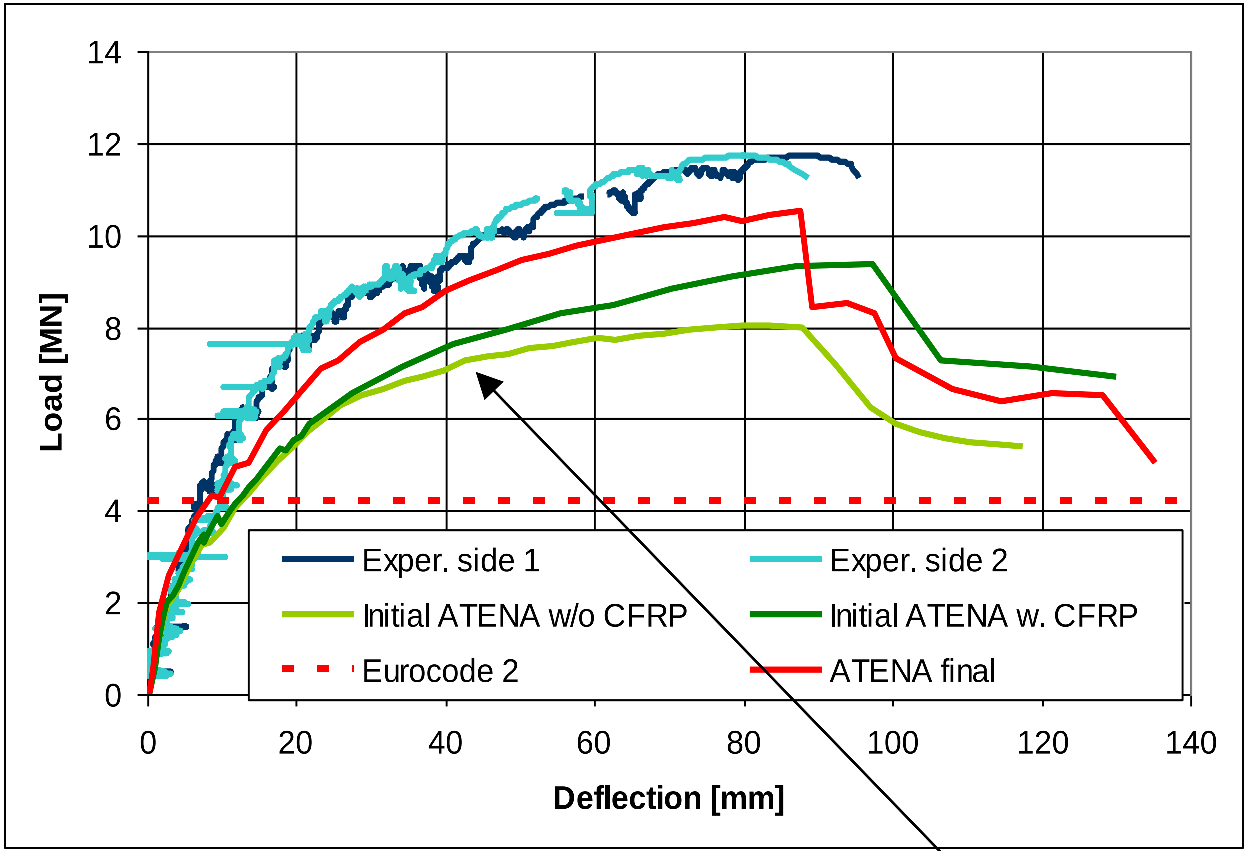
ATENA analysis



Field test

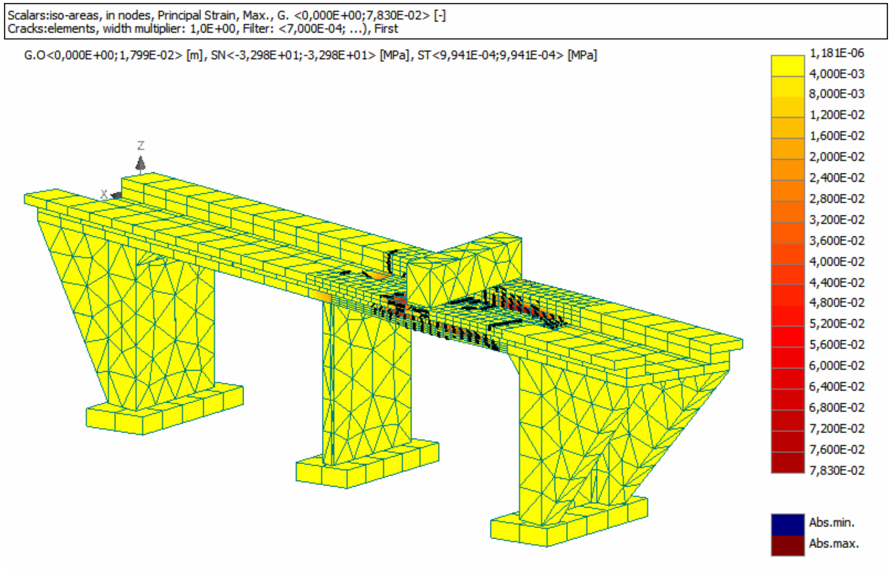
Örnsköldsvik Bridge – shear strength comparison, experiment, ATENA calculation

Analysis with stirrups



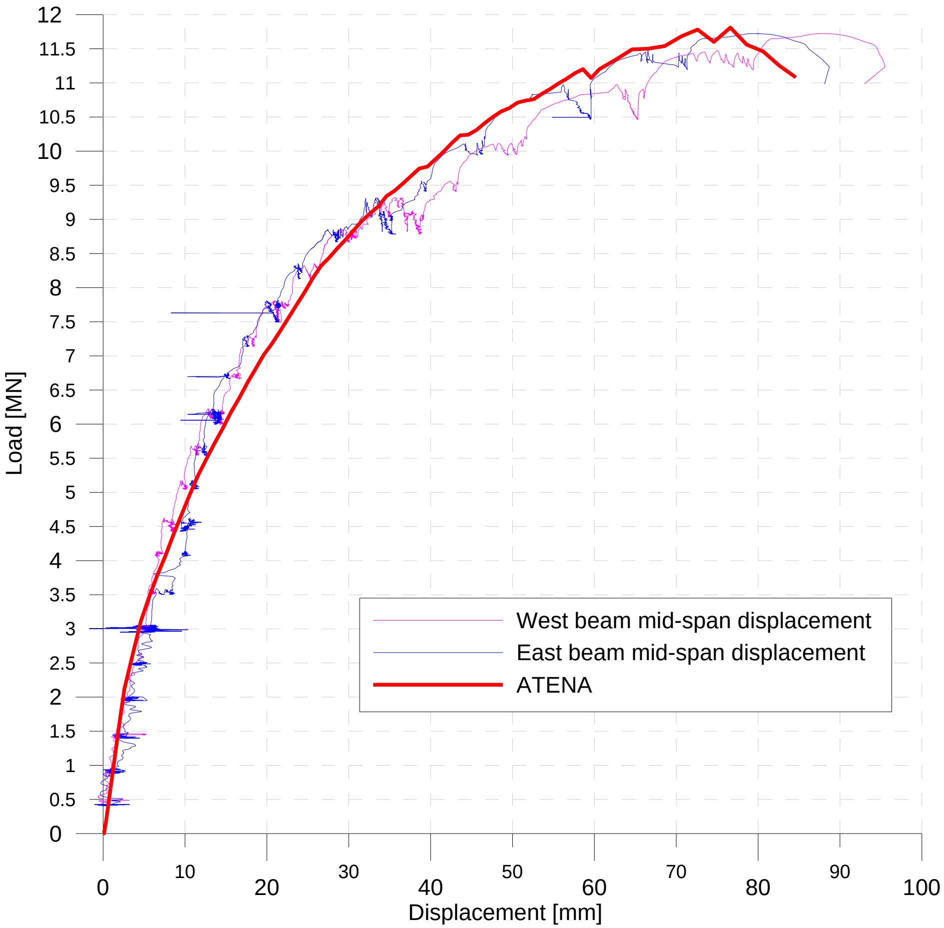
Stirrups not modelled in the initial analyses

3D analysis by LTU

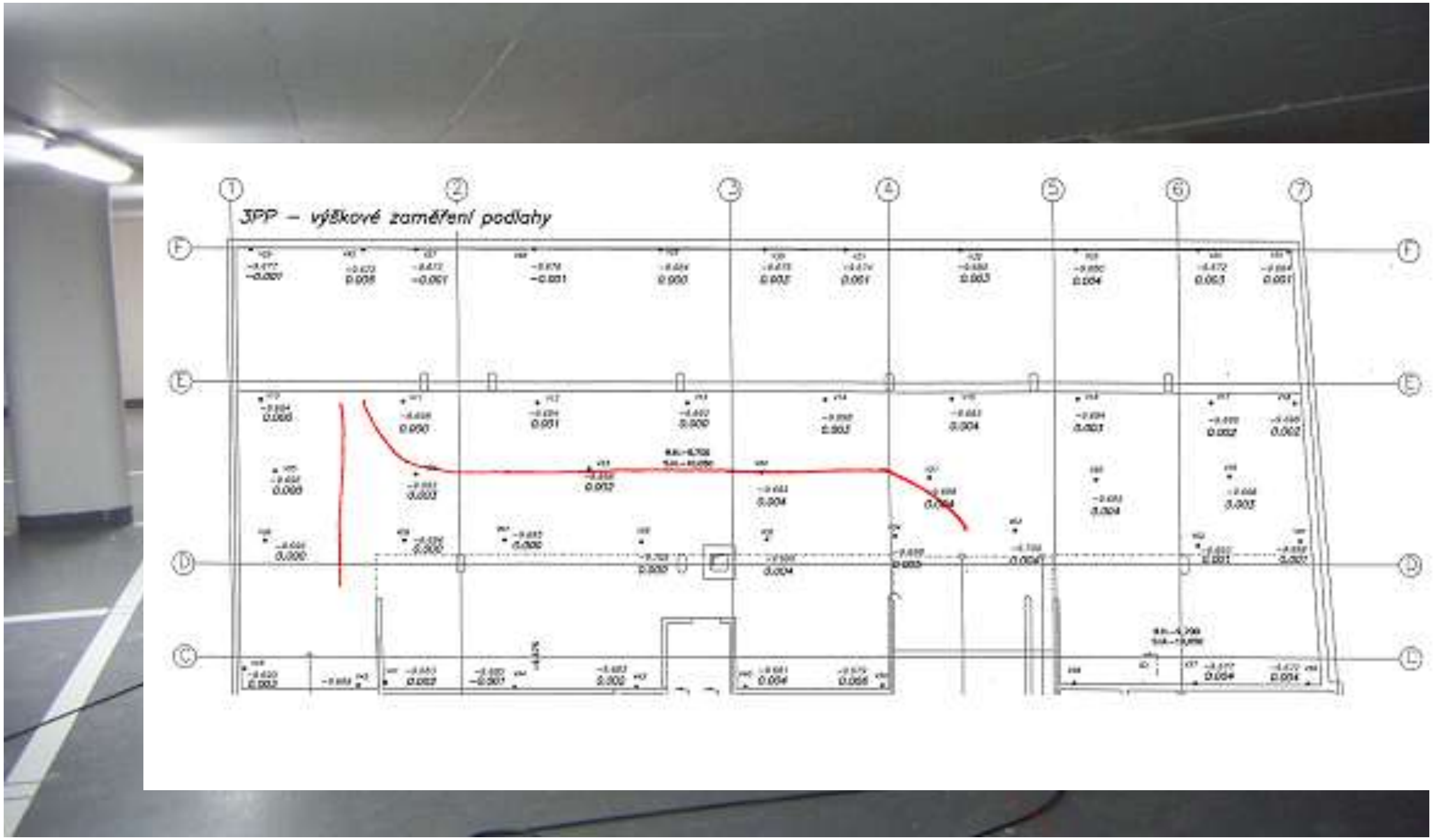


Scalars:iso-areas, in nodes, Principal Strain, Max., G. <0,000E+00;7,830E-02> [-]
Cracks:elements, width multiplier: 1,0E+00, Filter: <7,000E-04; ...), First

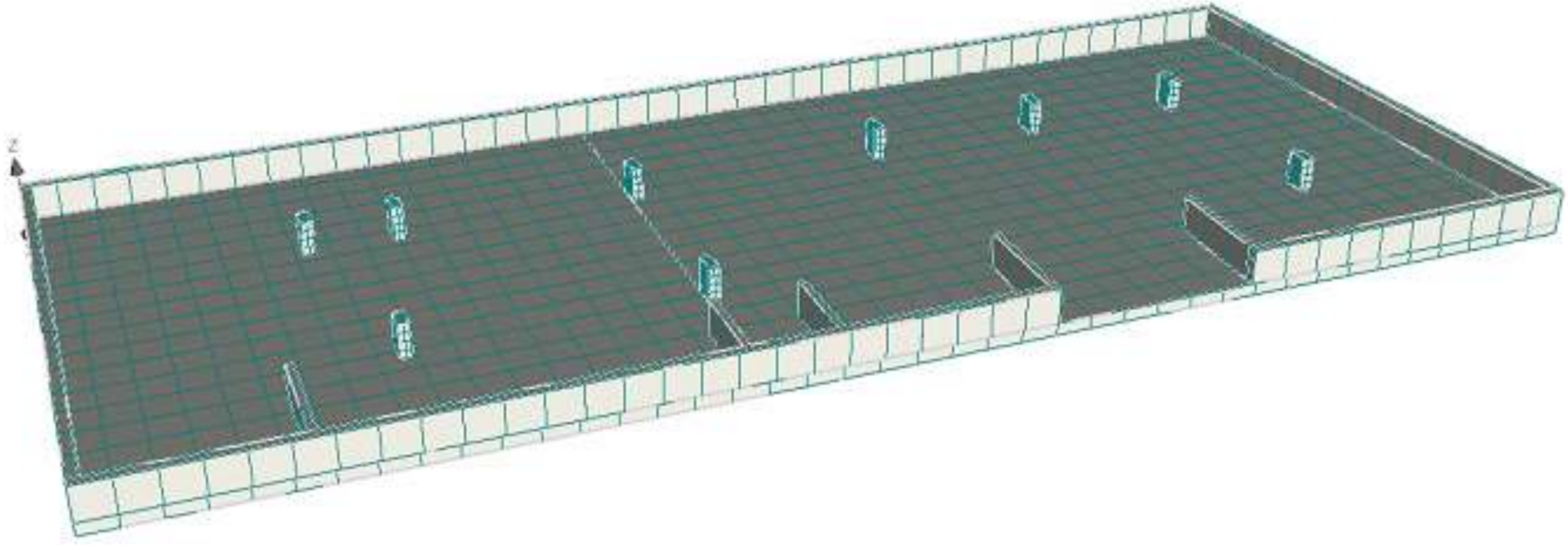
G.O<0,000E+00;1,799E-02> [m], SN<-3,298E+01;-3,298E+01> [MPa], ST<9,941E-04;9,941E-04> [MPa]



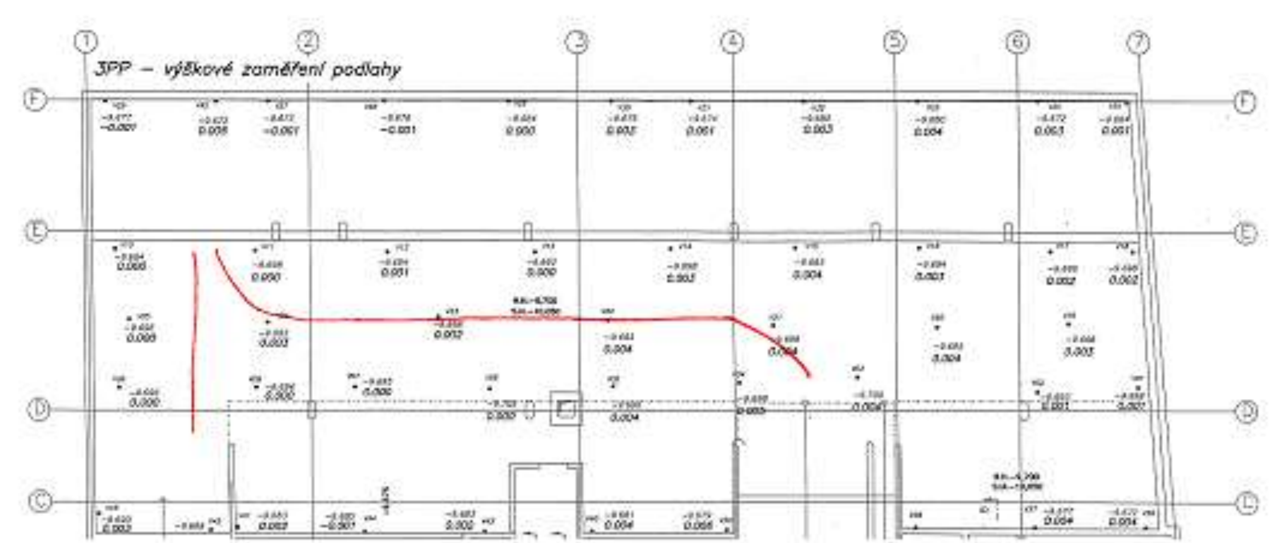
Foundation slab – cracks due to underground water pressure forensic investigation



ATENA numerical simulation



Foundation slab
Measured cracks



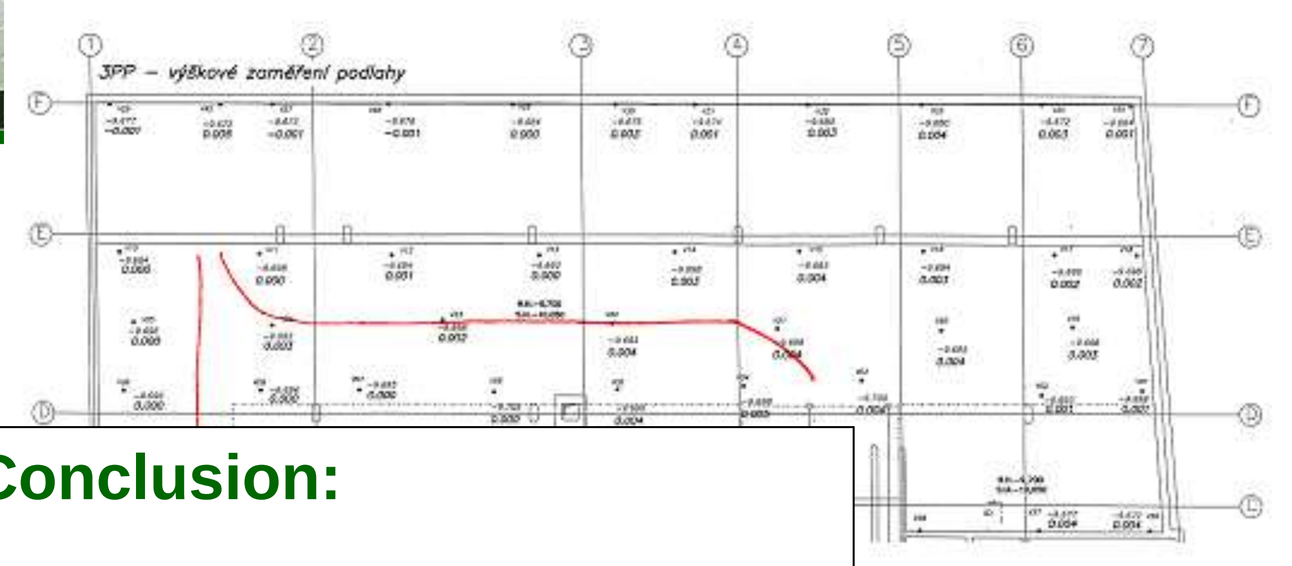
FE analysis

Results due to water pressure

Scalars:iso-areas, in nodes, Crack Width, Cod1, G. <-9.455E-04;2.577E-03> [m]
Cracks:elements, width multiplier: 1.0E+00, Filter: <3.000E-04; ...), First, Second, Third

G.O<0.000E+00;3.727E-03> [m], SN<-2.754E+00;-2.754E+00> [MPa], ST<6.136E-04;6.136E-04> [MPa]

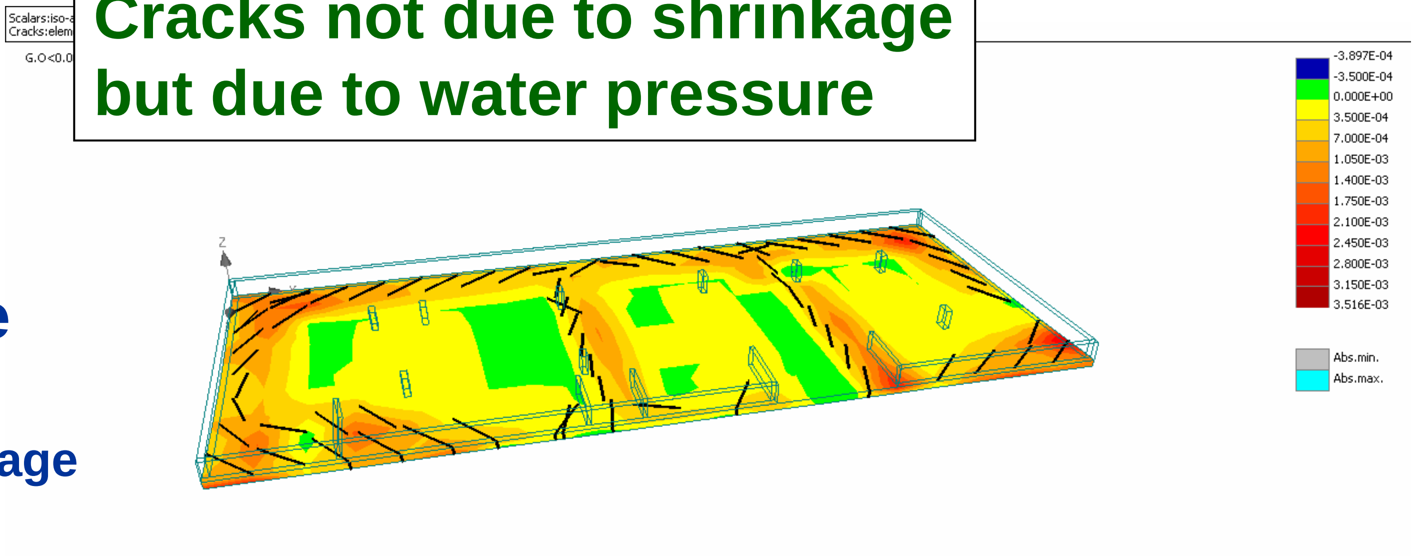
Foundation slab
Measured cracks



Conclusion:

Cracks not due to shrinkage but due to water pressure

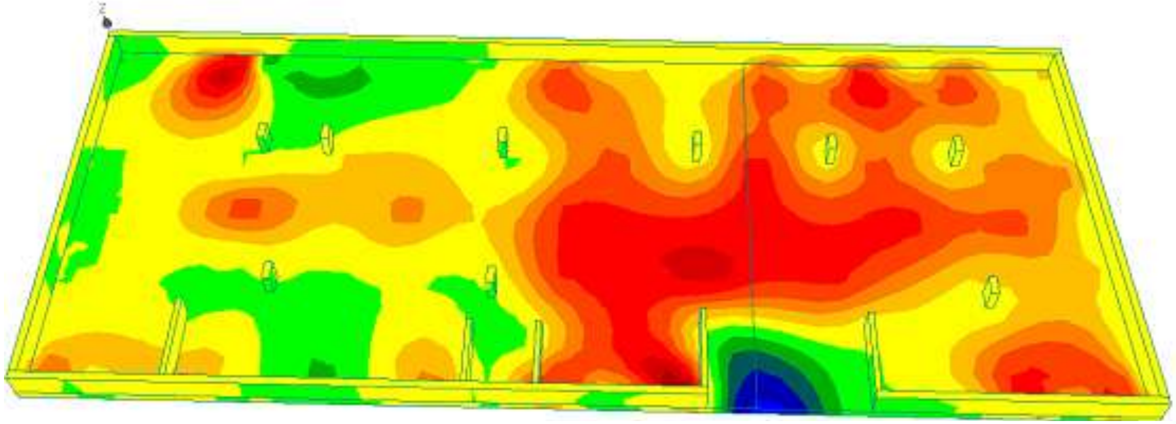
FE analysis



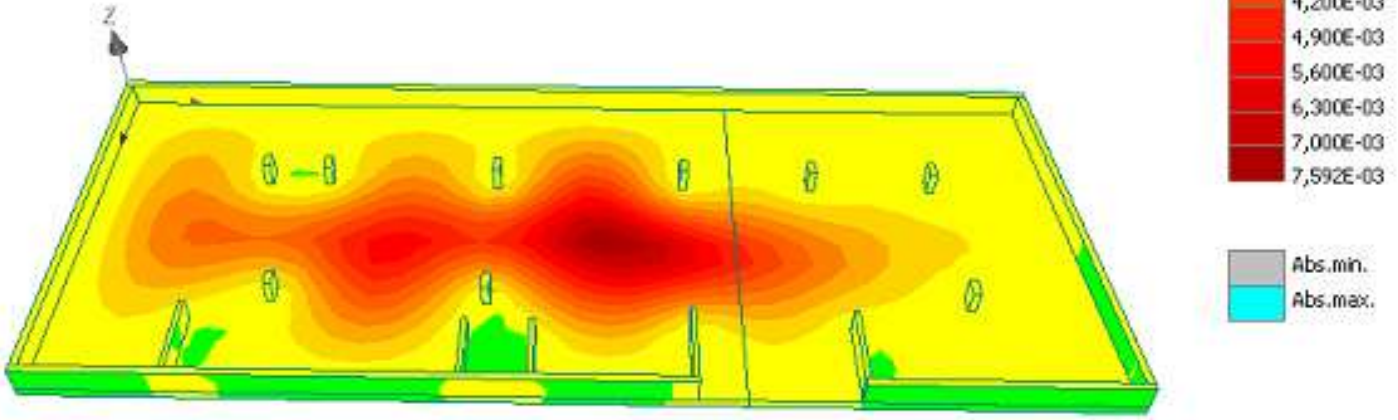
Results due to shrinkage

Constant shrinkage based on EC2

Measured deflections



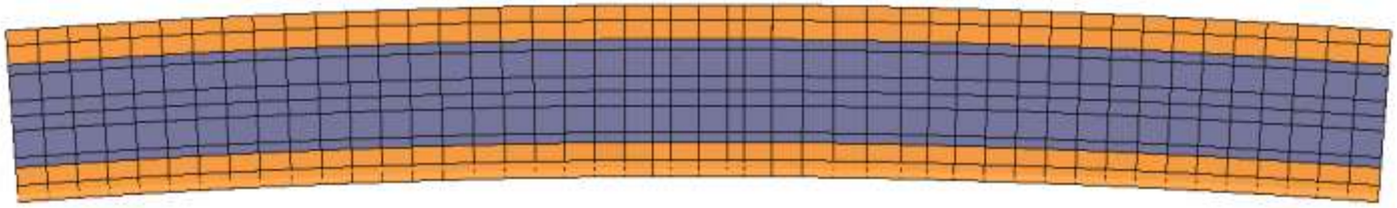
Analyzed deflections

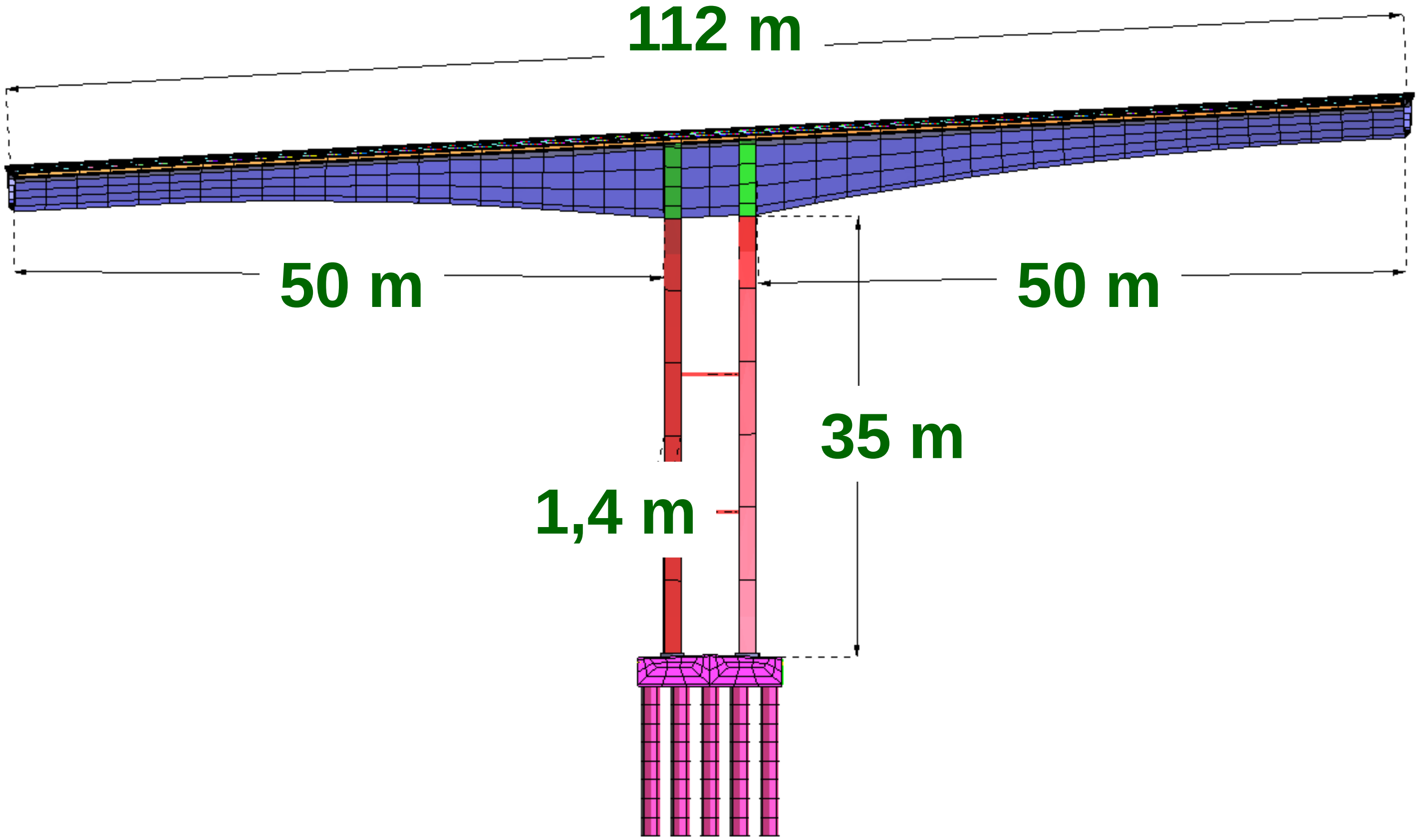


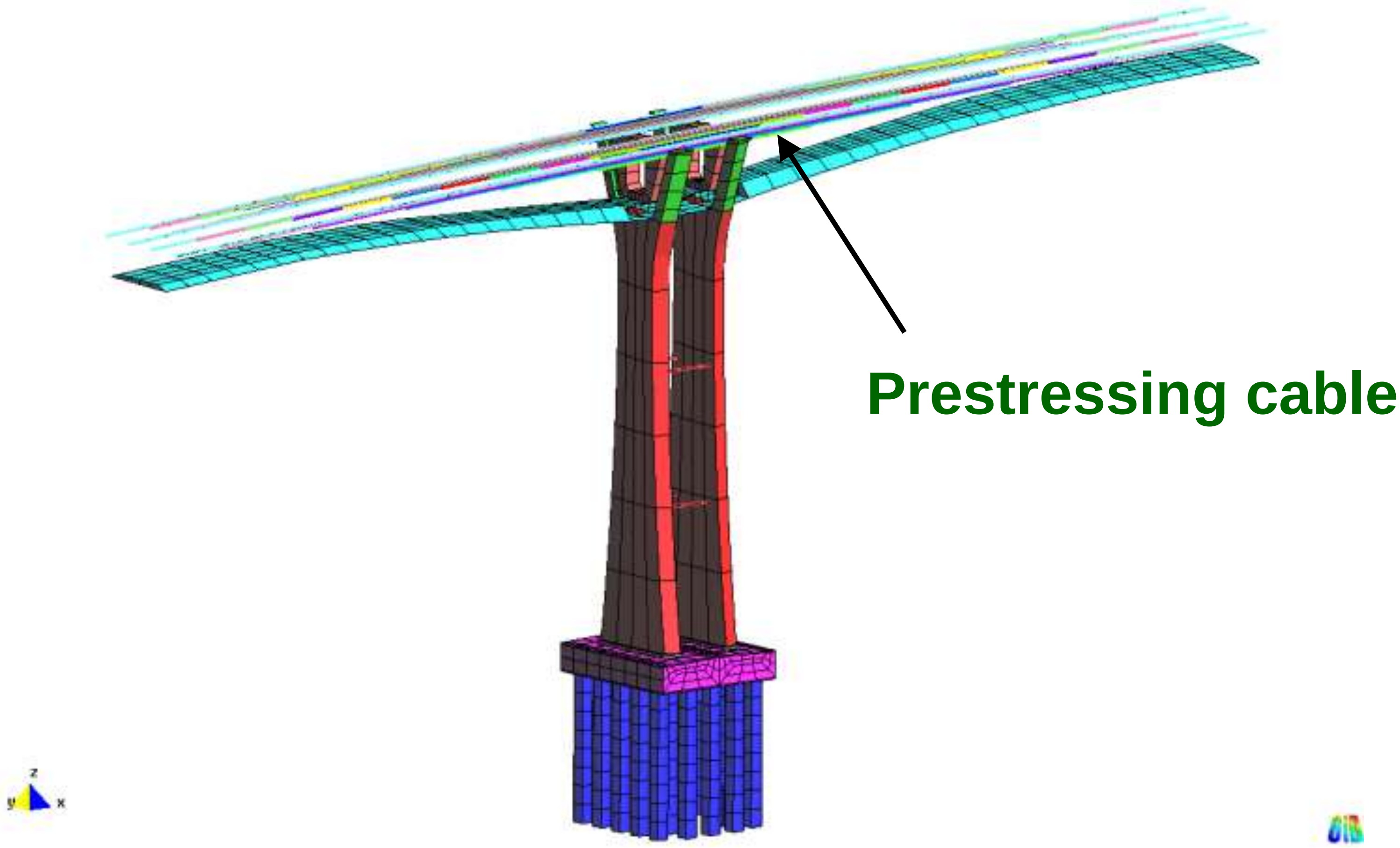
Construction 514, bridge crossing river Berounky near Prague, Czech Rep., design Novák & Partner, Ing. M. Šístek

Global verification of safety during construcion stages

Double console
Pier n. 39

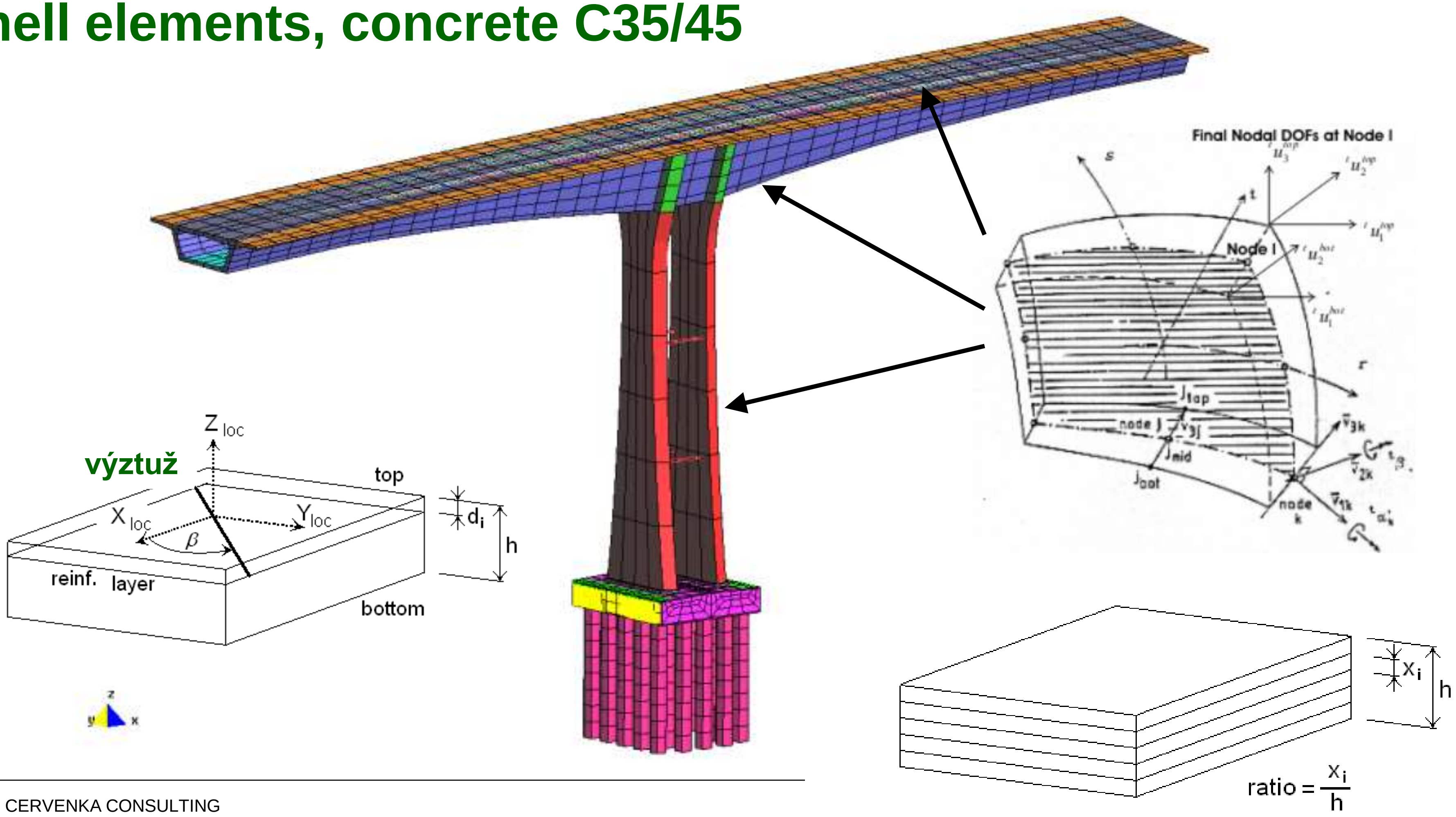




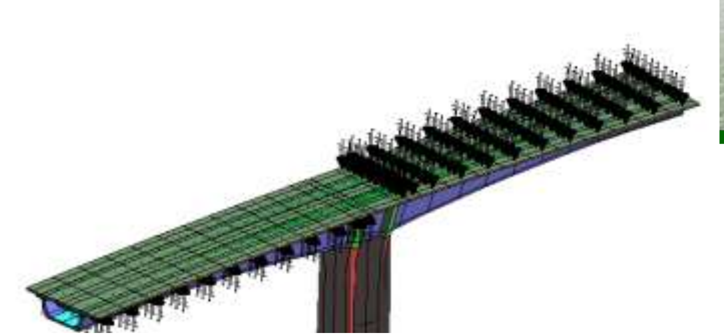


Prestressing cables

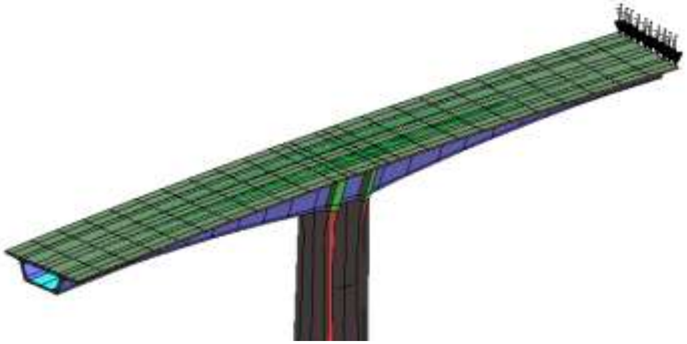
Special continuum 3D layered shell elements, concrete C35/45



Loading Cases



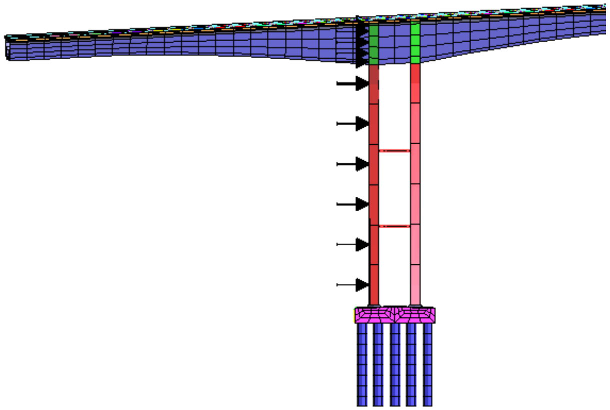
ZS16 vertical wind pressures

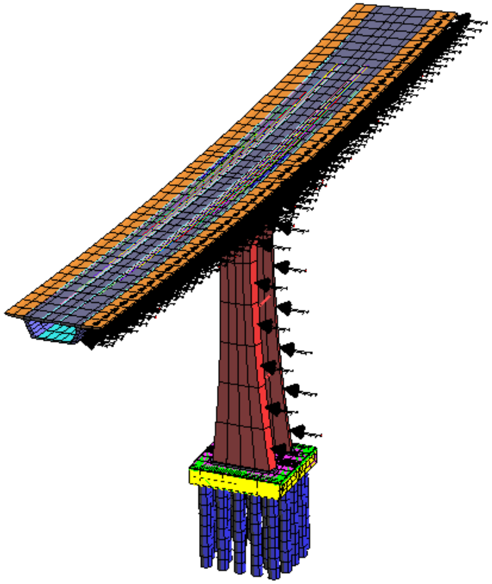


ZS17 concreting vehicle

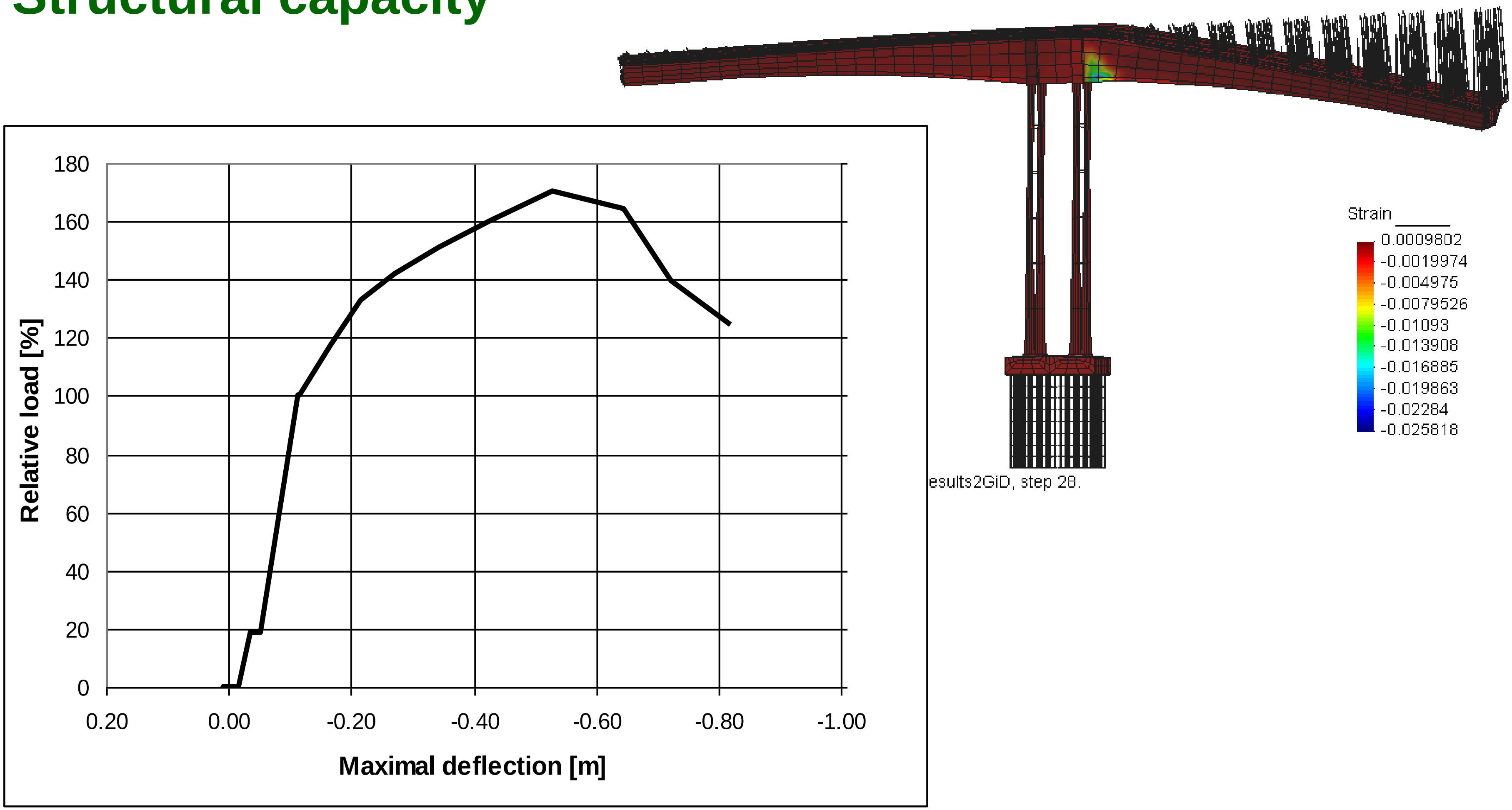
ZS18 longitudinal wind

ZS19 cross-wind





Structural capacity

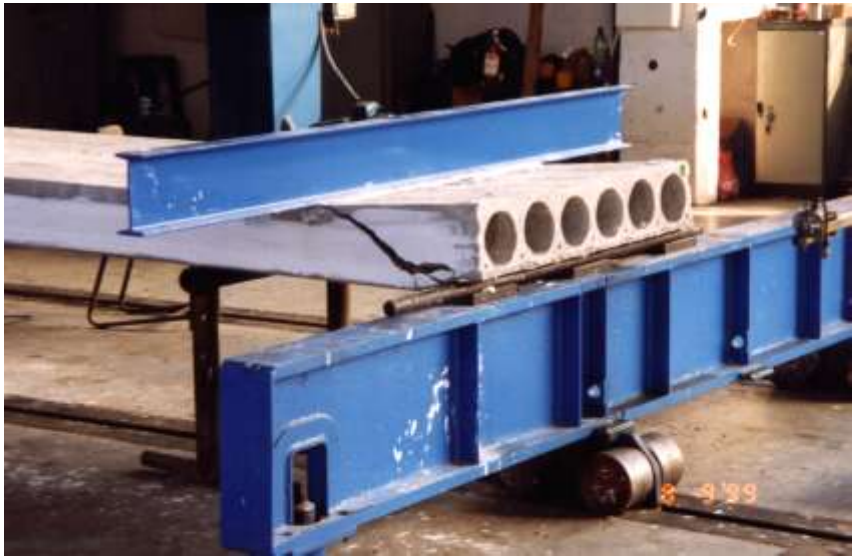


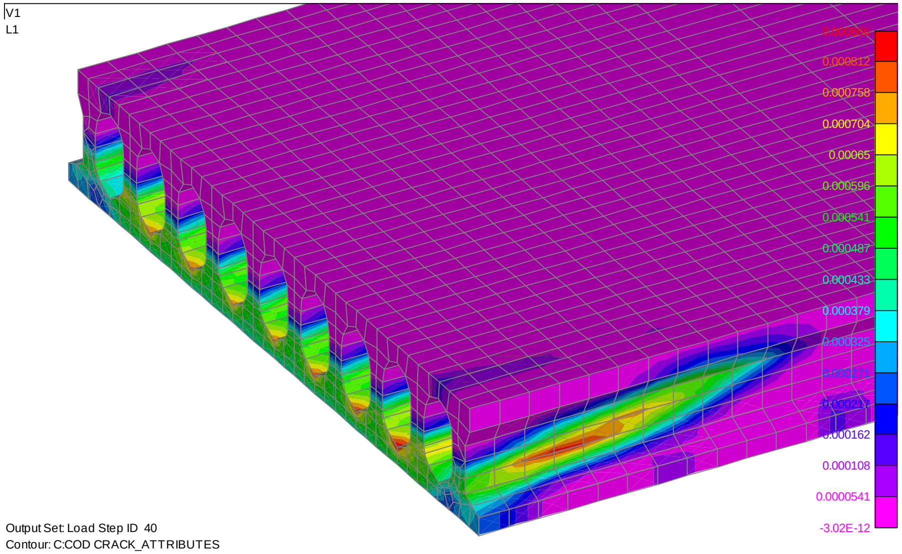
Optimization of precast structures

precast prestressed hollow core slabs without shear reinforcement

shear failure test in laboratory …

and in nonlinear computer simulation (crack widths)

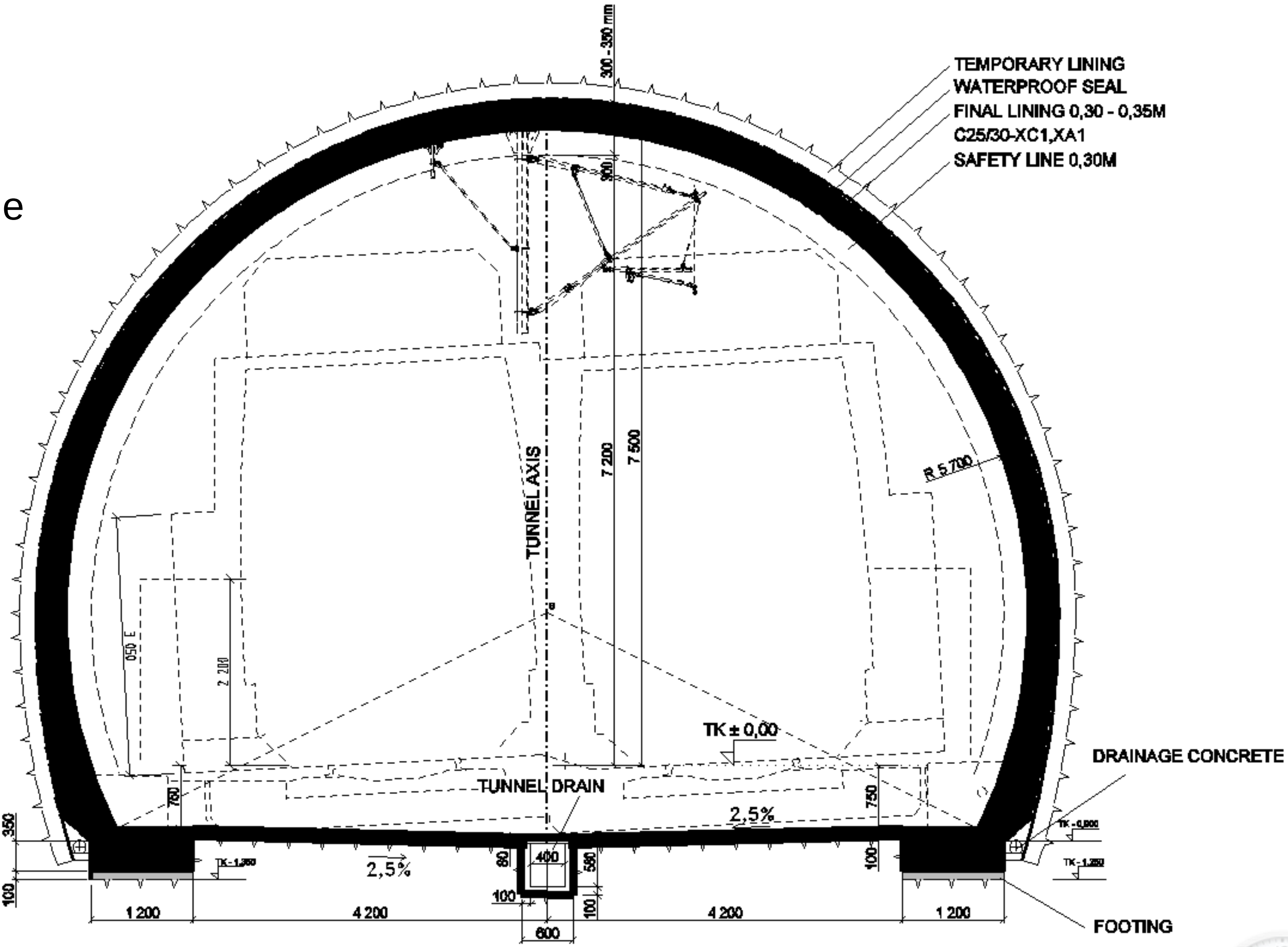




ATENA applications

plain concrete lining
railway tunnel in Prague

typical cross section
outer diameter 6 m



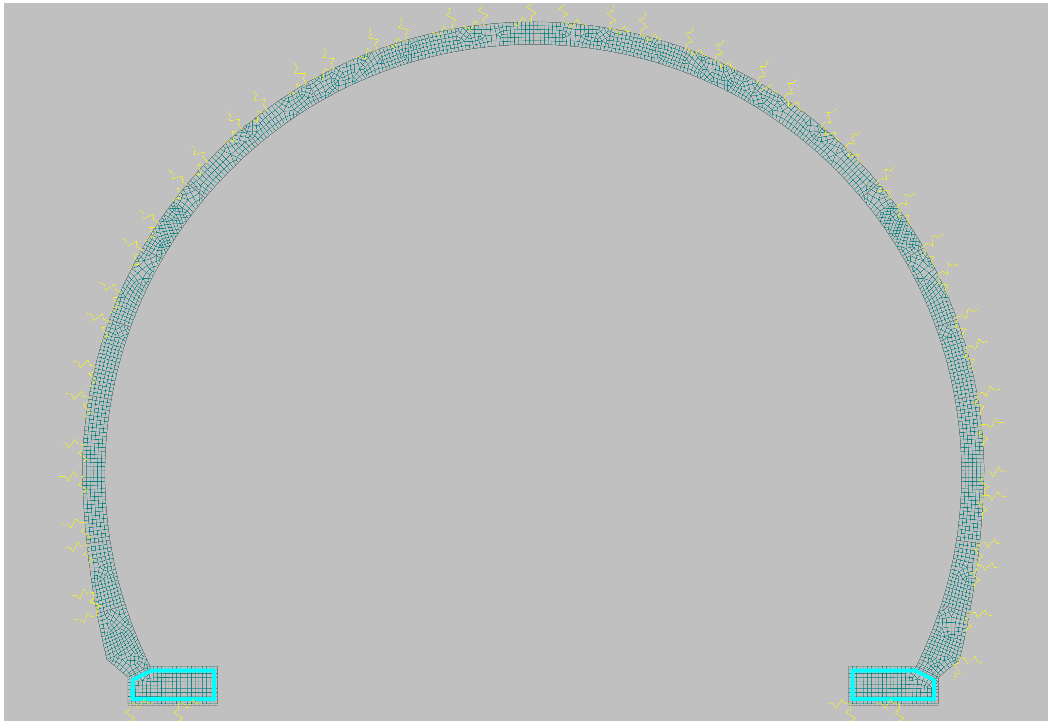
New Railway Connection in Prague

New Railway Connection in Prague – tunnels under the Vítkov Hill

Nonlinear analysis of the tunnel profile

finite element model
    5000 elements

1 m longitudinal section
    plane stress state
supported by nonlinear springs
    reflect soil properties

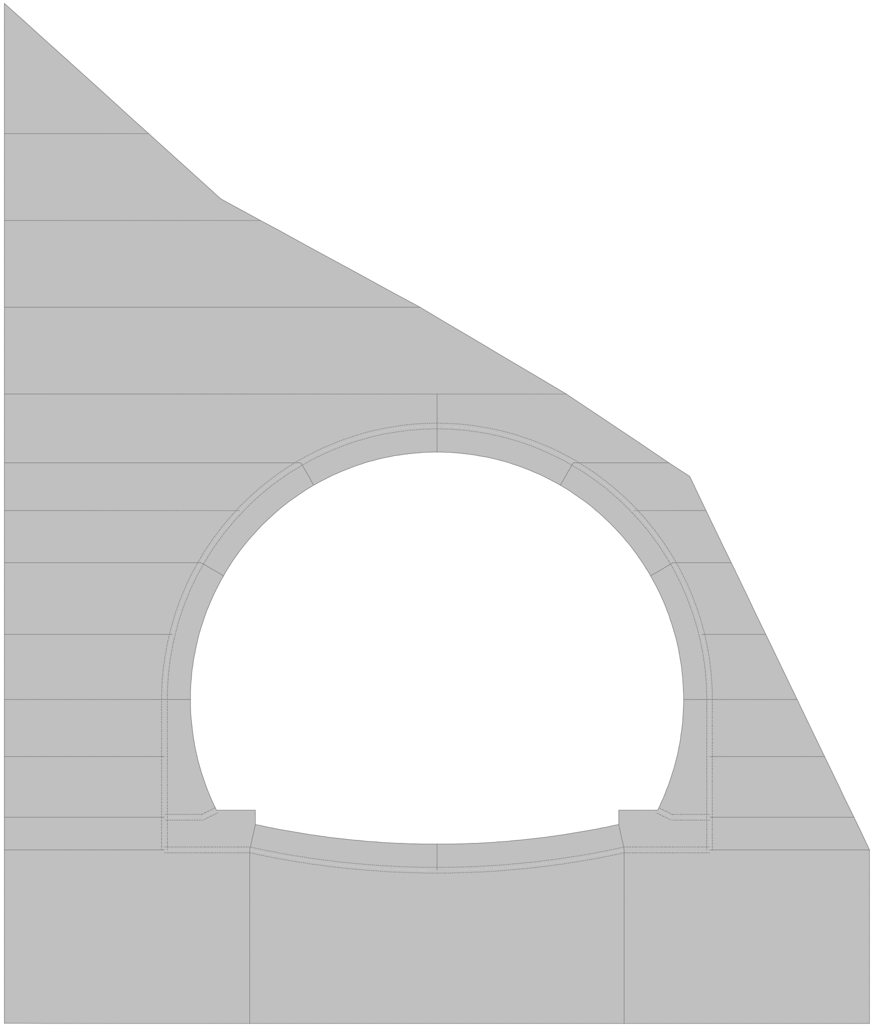


variants:
    various upper vault thickness
    plain or reinforced
    with or without bottom vault

Nonlinear analysis of the tunnel profile

finite element model

1 m longitudinal section
    plane stress state
supported by nonlinear springs
    reflect soil properties



Drucker-Prager ground

variants:
- various upper vault thickness
- plain or reinforced
- with or without bottom vault

Results from NLA

Iso-areas of

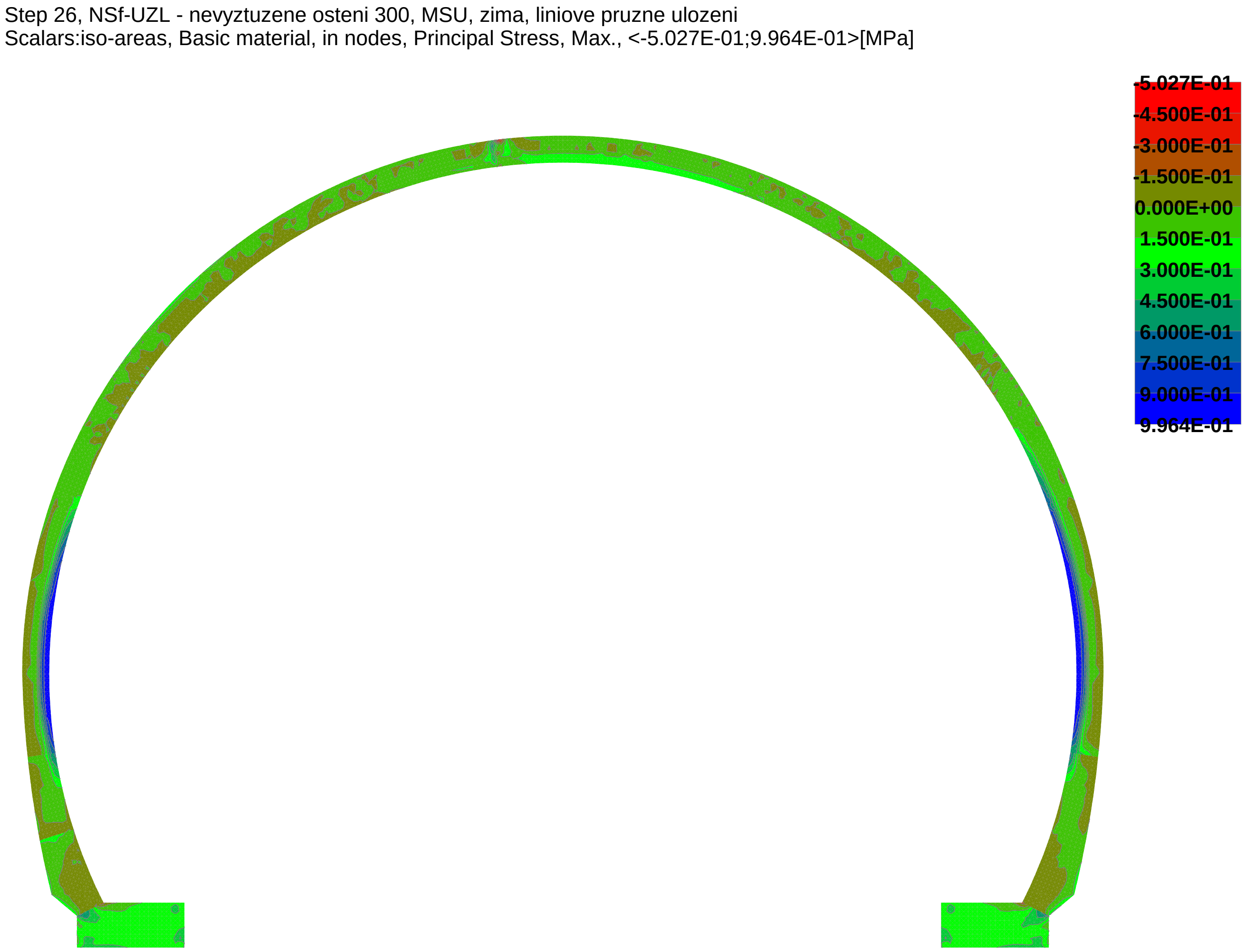
principal stress

maximal (tensile)

unreinforced

ultimate limit state

dead load
creep
shrinkage
temperature in winter



Step 26, NSf-UZL - nevyztuzene osteni 300, MSU, zima, liniove pruzne ulozeni
Scalars:iso-areas, Basic material, in nodes, Principal Stress, Max., <-5.027E-01;9.964E-01>[MPa]

Results from NLA

Step 26, NSf-UZL - nevyztuzene osteni 300, MSP, zima, liniove pruzne ulozeni
Cracks: in elements, openning: <-1.544E-04;1.592E-03>[m], Sigma_n: <-1.237E+00;9.998E-01>[MPa], Sigma_T : <1.017E-16;5.182

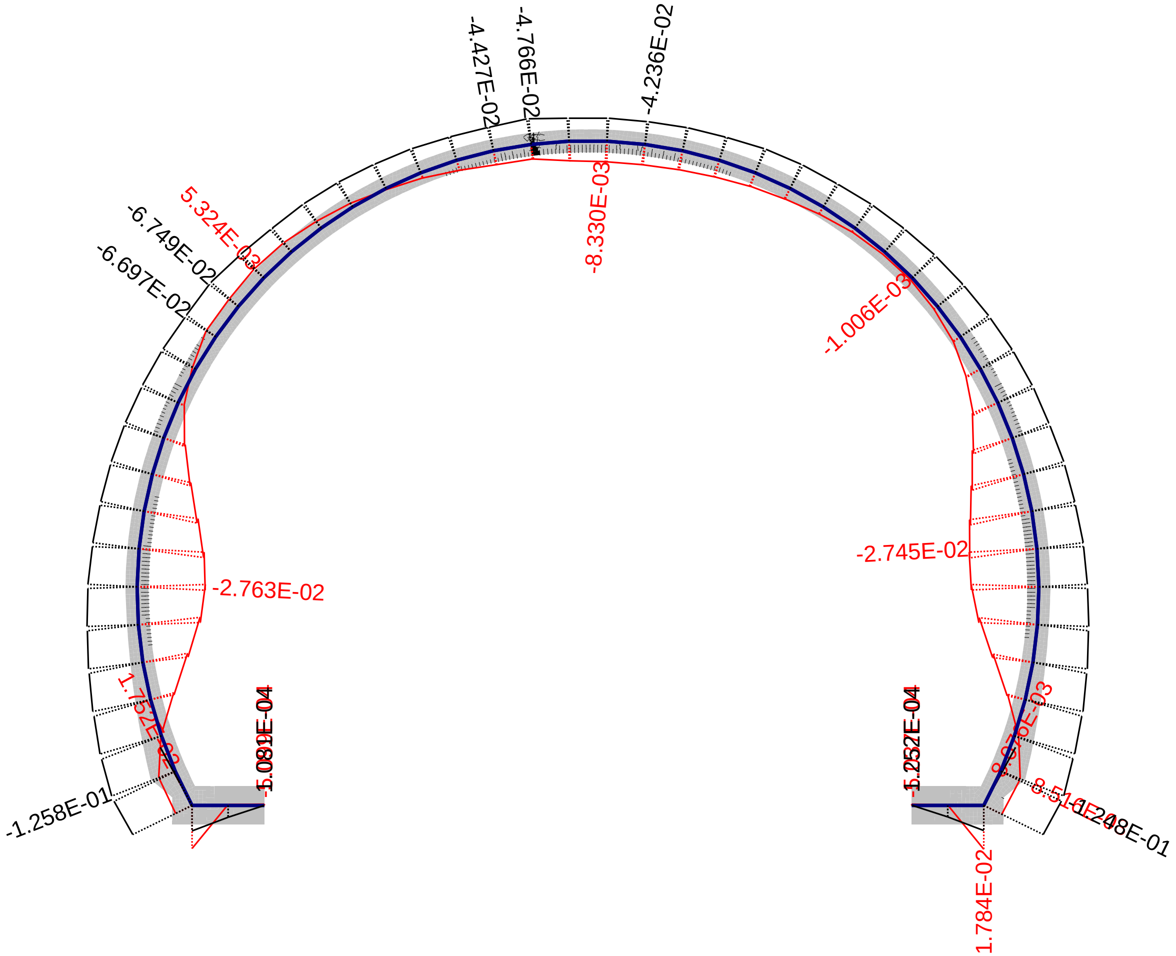
normal forces

bending moments

unreinforced

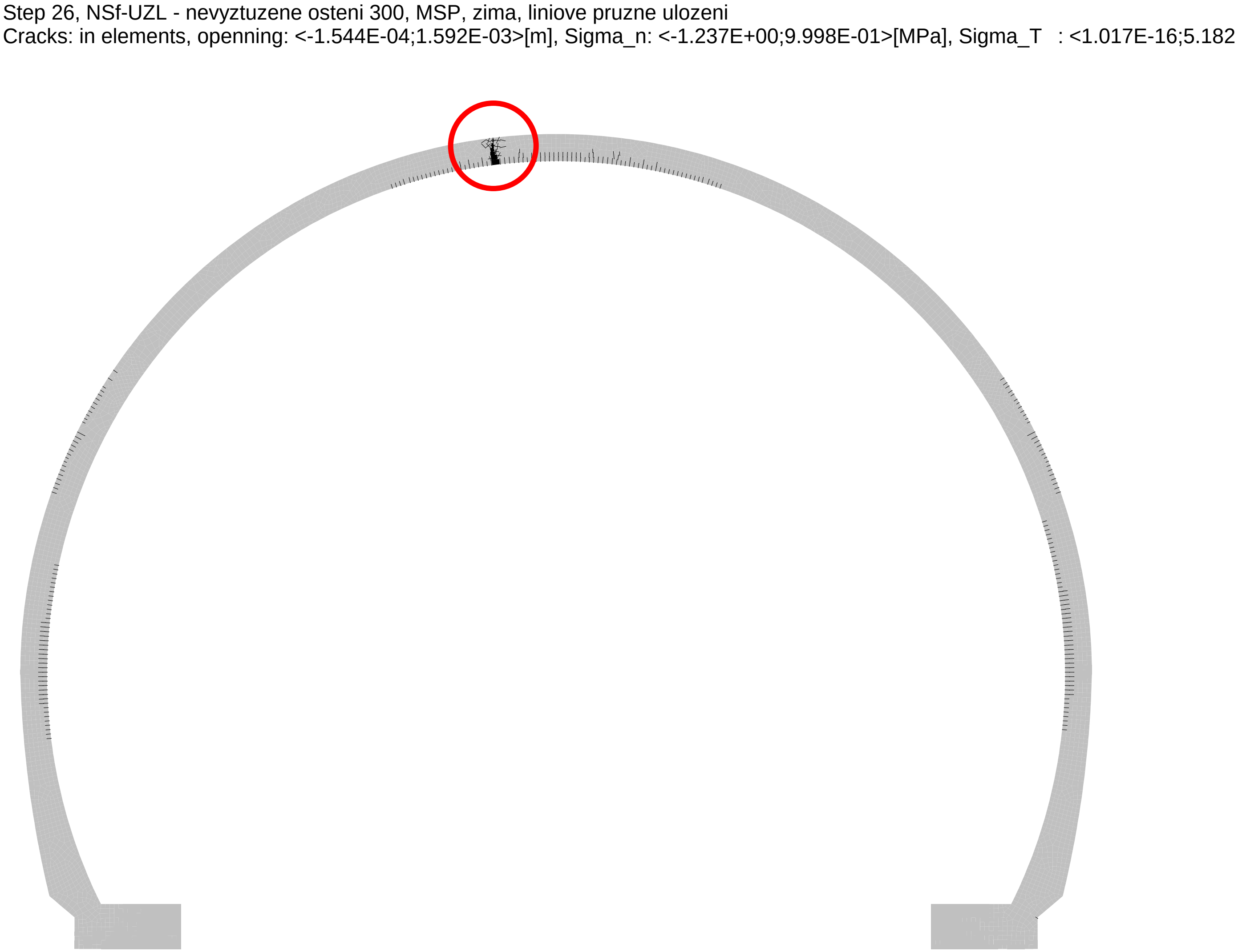
ultimate limit state

dead load
creep
shrinkage
temperature in winter



Results from NLA

Crack pattern



Step 26, NSf-UZL - nevyztuzene osteni 300, MSP, zima, liniove pruzne ulozeni
Cracks: in elements, openning: <-1.544E-04;1.592E-03>[m], Sigma_n: <-1.237E+00;9.998E-01>[MPa], Sigma_T : <1.017E-16;5.182

unreinforced

ultimate limit state

dead load
creep
shrinkage
temperature in winter

Results from NLA

Main crack

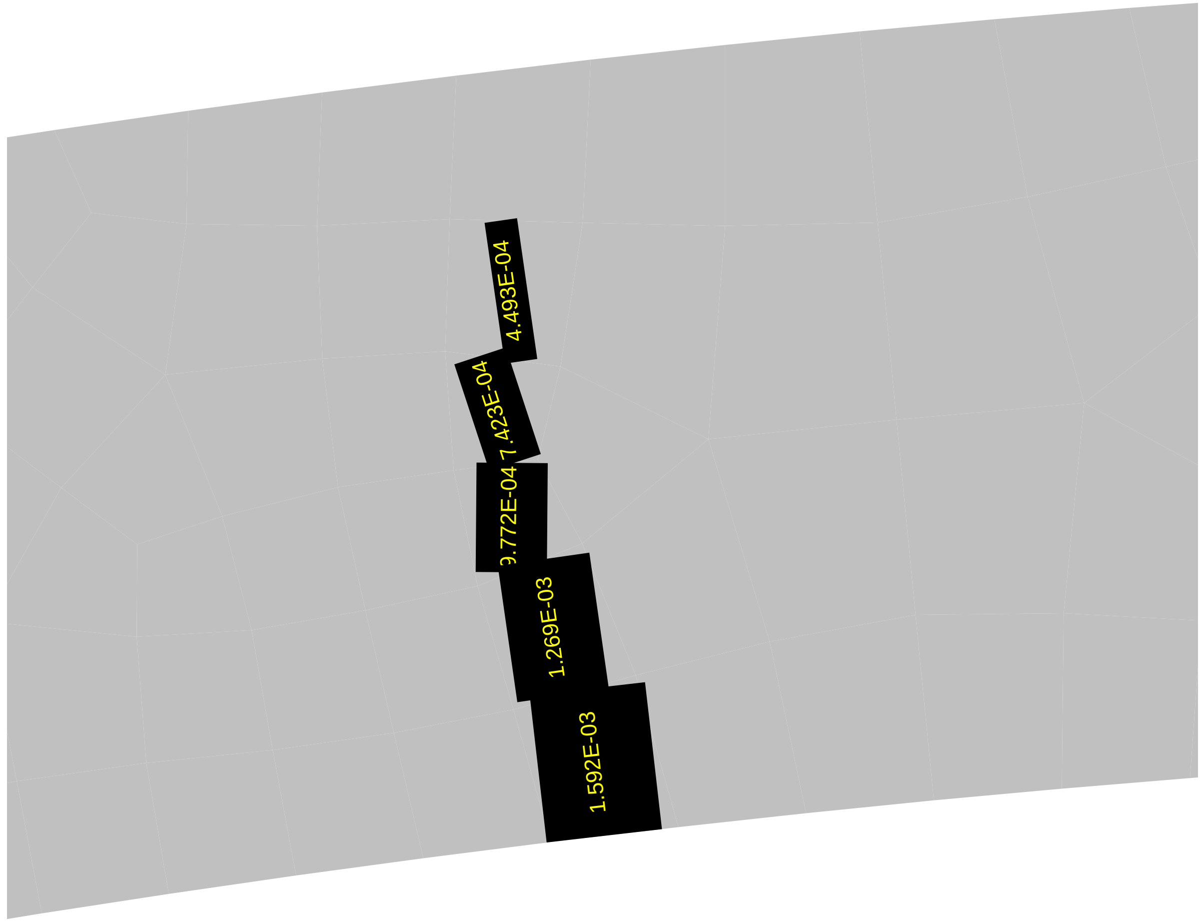
description
of crack width
max. 1.6 mm

unreinforced

ultimate limit state

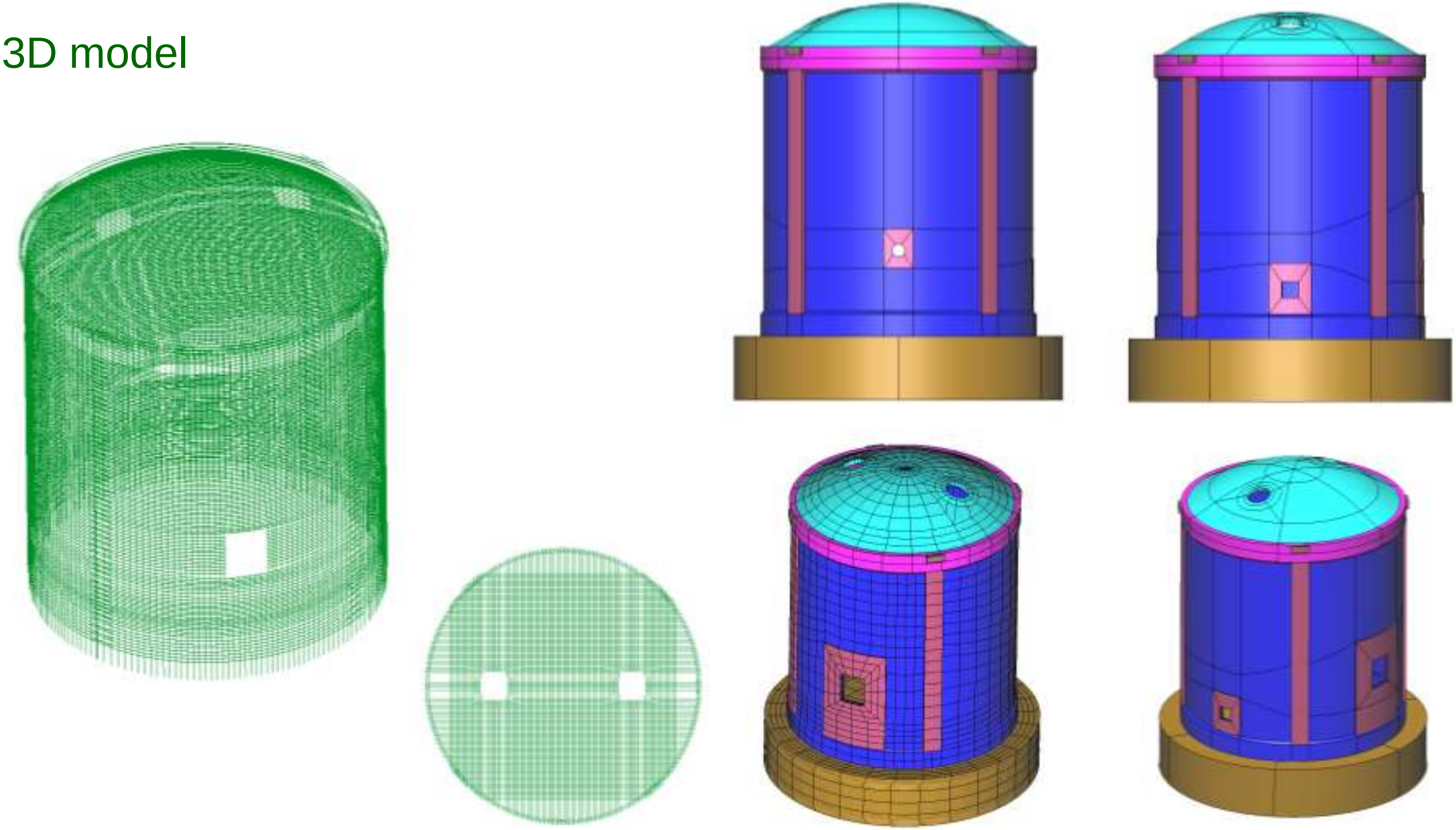
dead load
creep
shrinkage
temperature in winter

Step 26, NSf-UZL - nevyztuzene osteni 300, MSP, zima, liniove pruzne ulozeni
Cracks: in elements, <2.000E-04; ...), openning: <-1.544E-04;1.592E-03>[m], Sigma_n: <-1.237E+00;9.998E-01>[MPa], Sigma_T : <

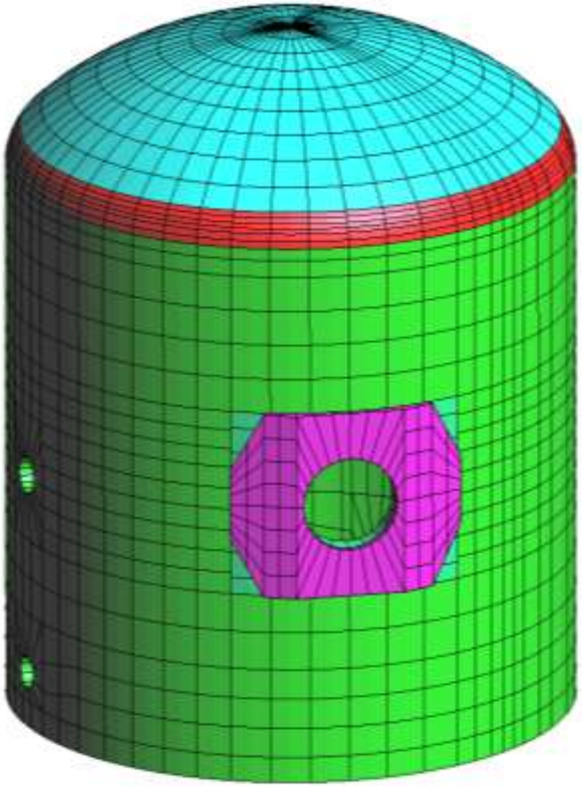


BARC, Indie, Containment Pressure Test Model 1:4

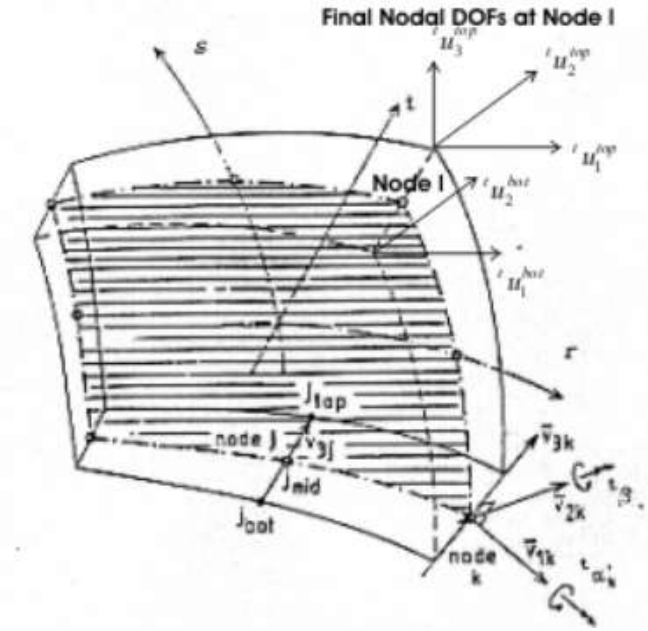
3D model



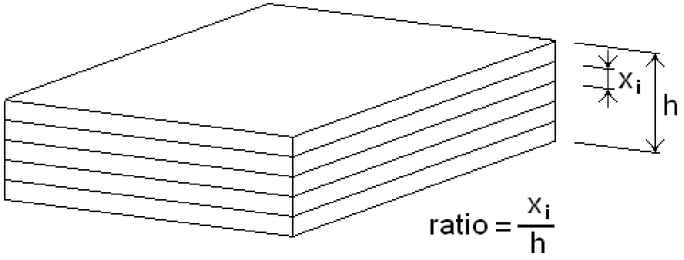
ATENA
3D shell element



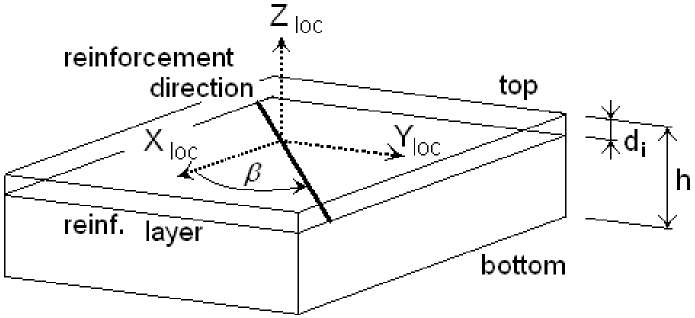
geometry



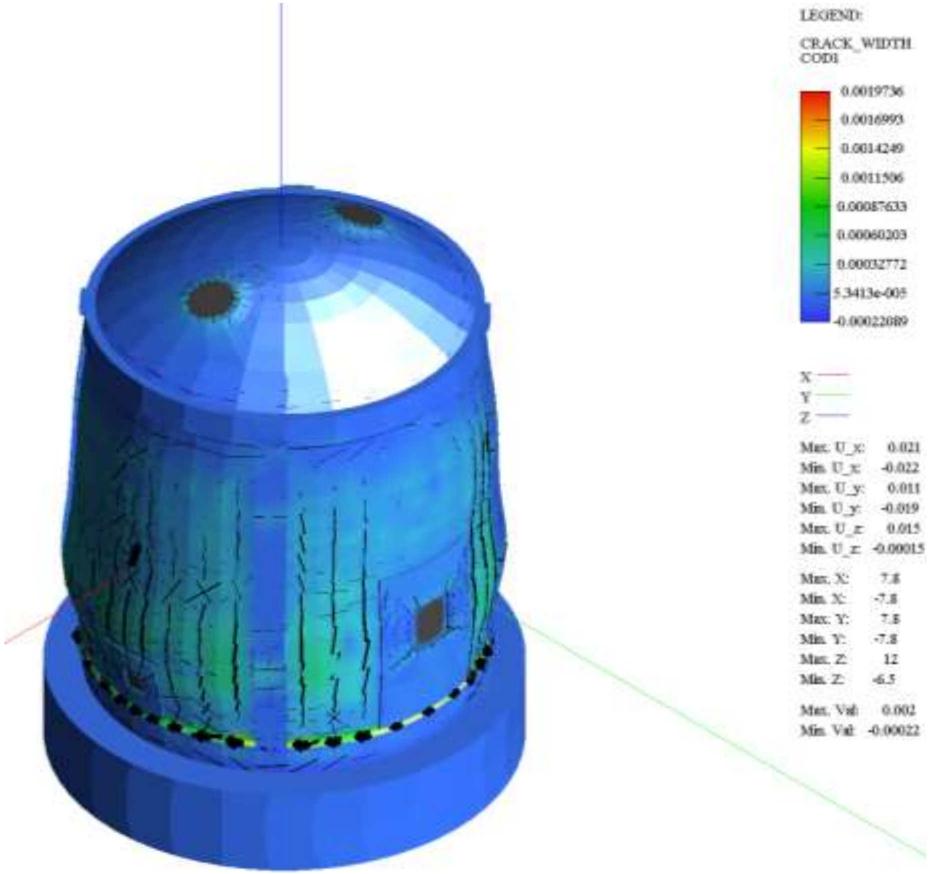
layers

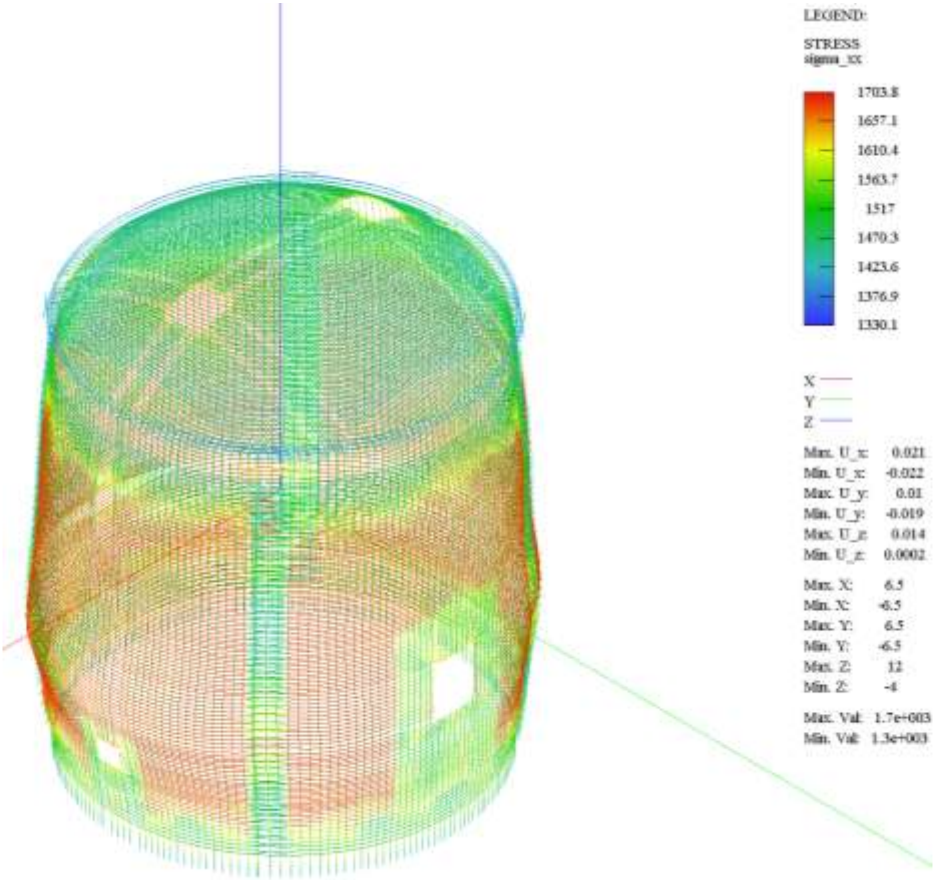


reinforcement

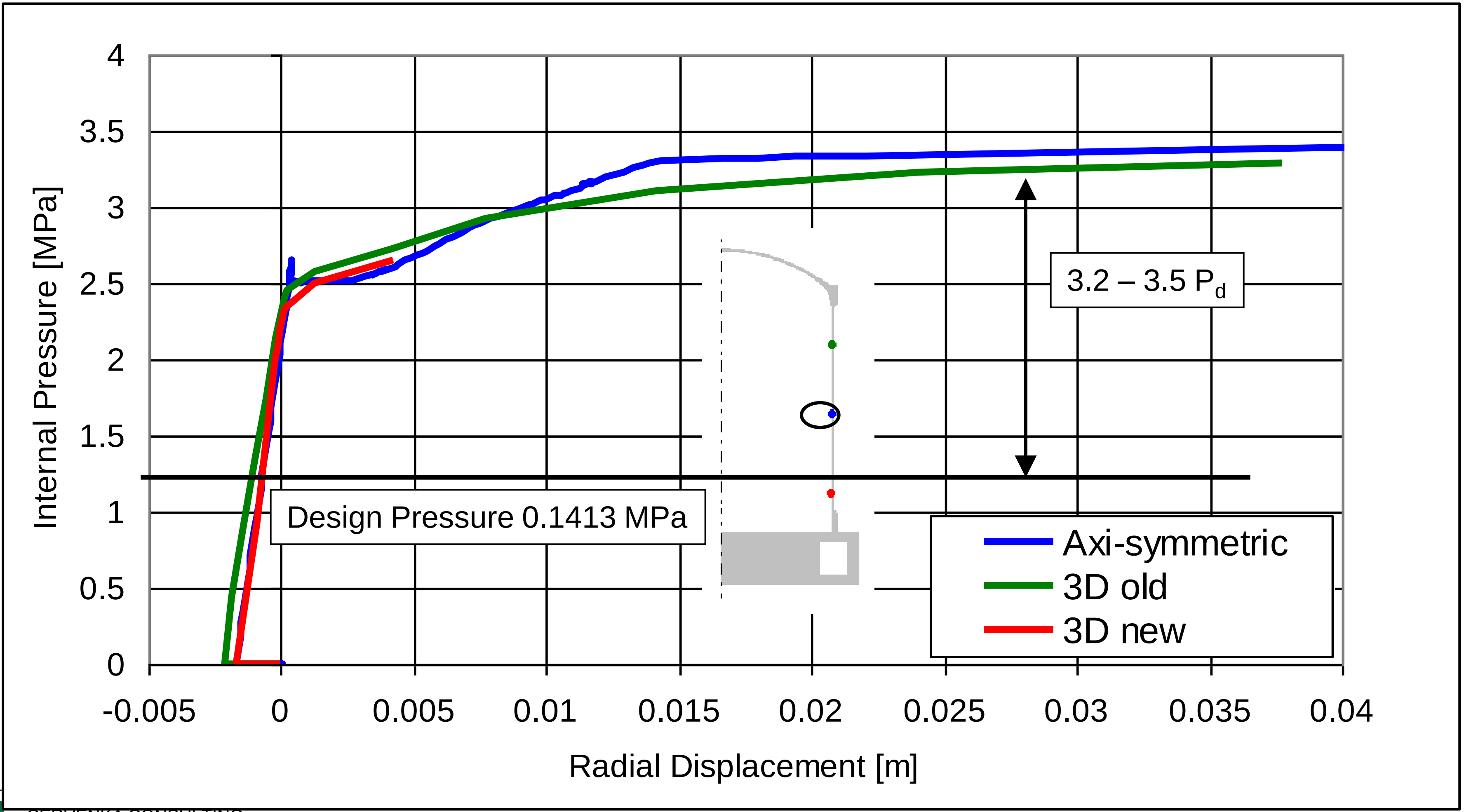


3D Analysis – 3.00 design pressure, P = 0.4239 MPa

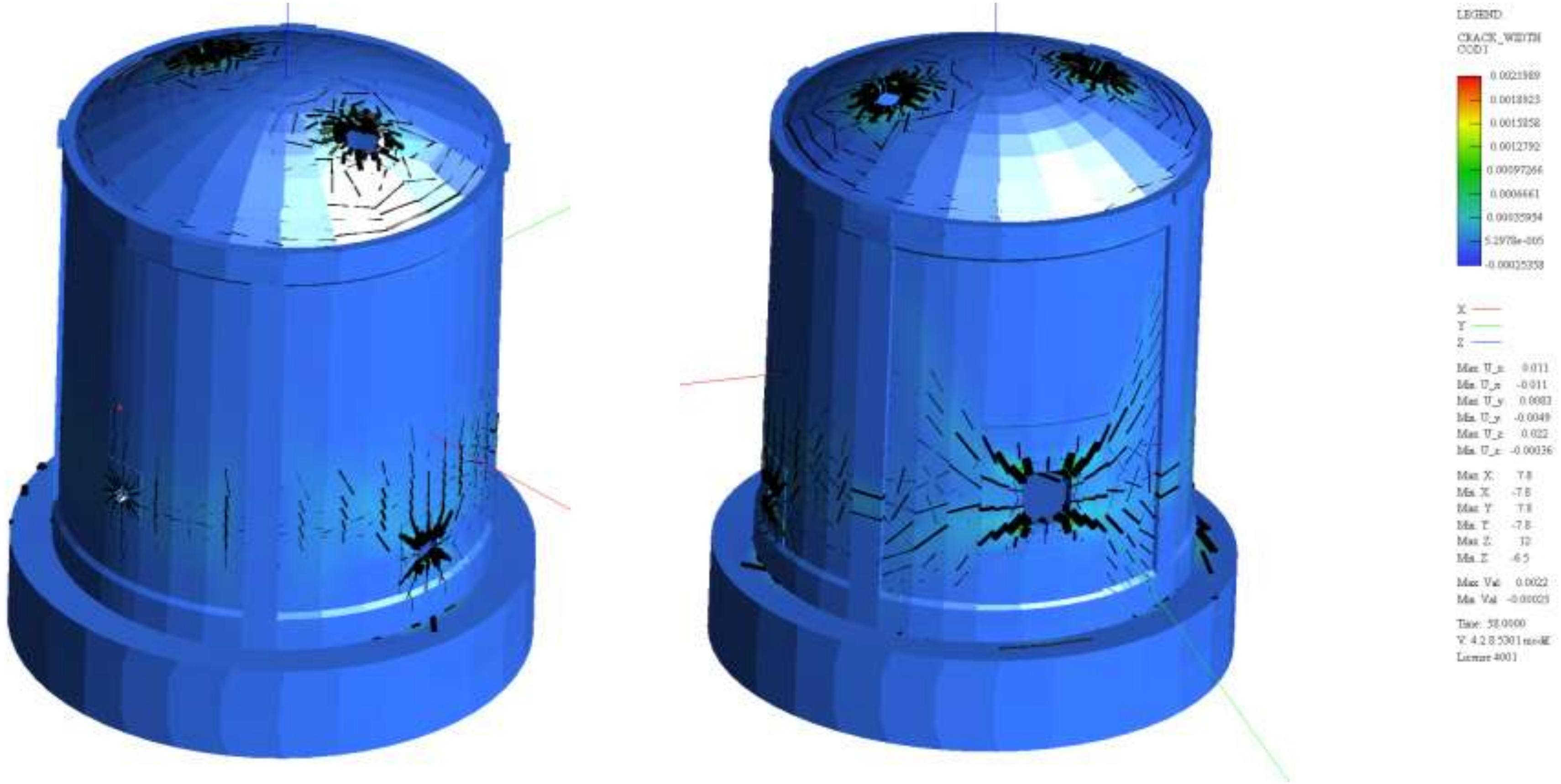




2D - 3D Analysis – Comparison



New 3D Model based on BARCOM 2009 workshop – corrected cover of openings



Safety formats for non-linear analysis - 4 methods

Example:

- **bending**
- **shear - deep beam**
- **bridge pier – geometric nonlin.**
- **railway tunnel**

Comparative study of different safety formats

Safety Formats for Nonlinear Analysis

$$E_d < \frac{R_x}{\gamma_R}$$

R_x - is the structural resistance obtained by nonlinear analysis

g_R - is the global safety factor of the structural resistance

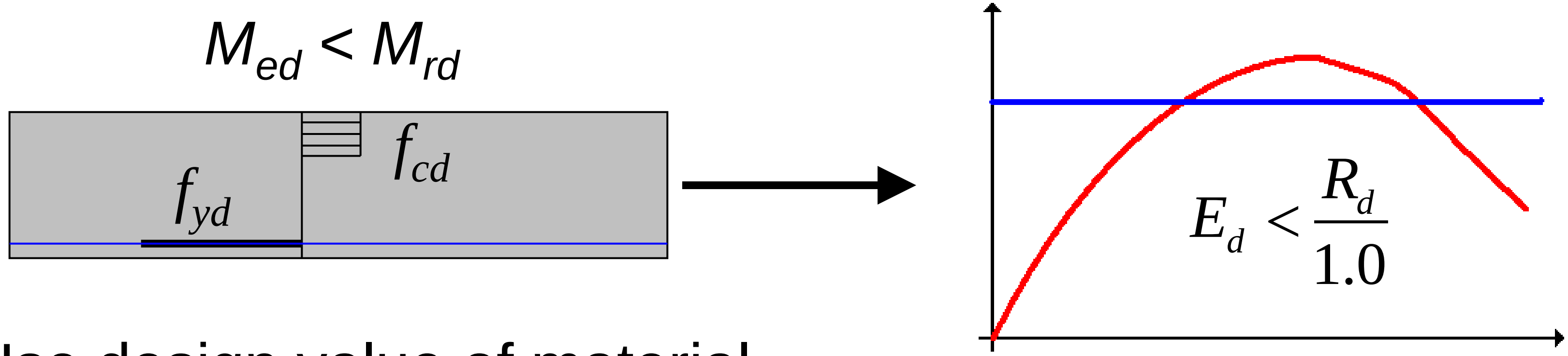
E_d - is the factorized load effect as in the case of partial safety factor method

Safety Format (1), PSF Method

Action Resistance

$$E_d(\gamma_{G,i}) \le R_d(\gamma_{d,j})$$

Partial safety factors



Use design value of material parameter to calculate R_d:

$$\text{design val.} = \frac{\text{characteristic val.}}{\text{partial safety factor}}$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1.5} = 20\ MPa$$

Safety Format (2), EN1992-2

Global safety factor

$$E_d(\gamma_{G,i}) \leq R_m / \gamma_R$$

All failure modes: $\gamma_R = 1.27$

Adjusted “mean” values of material parameters:

$$f_{cm} = \underset{\uparrow}{0.85} f_{ck} \qquad f_{ym} = 1.1 f_{yk}$$

1.1 x 1.15/1.5

Safety Format (3), ECOV
Estimate of Coefficient of Variation

Coefficient of variation, assuming lognormal distribution of resistance

Global resistance factor

we need
2 analyses

$$R_k = R_m \exp(-1.65\, V_R)$$

$$R_d = R_m \exp(-\alpha_R \beta\, V_R)$$

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right)$$

$$\gamma_R^m = \exp(\alpha_R \beta V_R)$$

$$\beta = 4.7$$

Reliability index

$$\alpha_R = 0.8$$

Resistance sensitivity

Safety Format (4), Probabilistic Analysis

Probability of failure:

(1) $P(E > R)$

(2) $Z = E - R$

$P(Z < 0)$ $\sim 10^{-6}$

Reliability index: $\beta(Z)$ ~ 4.7

EN 1990: Basis of structural design, 2002

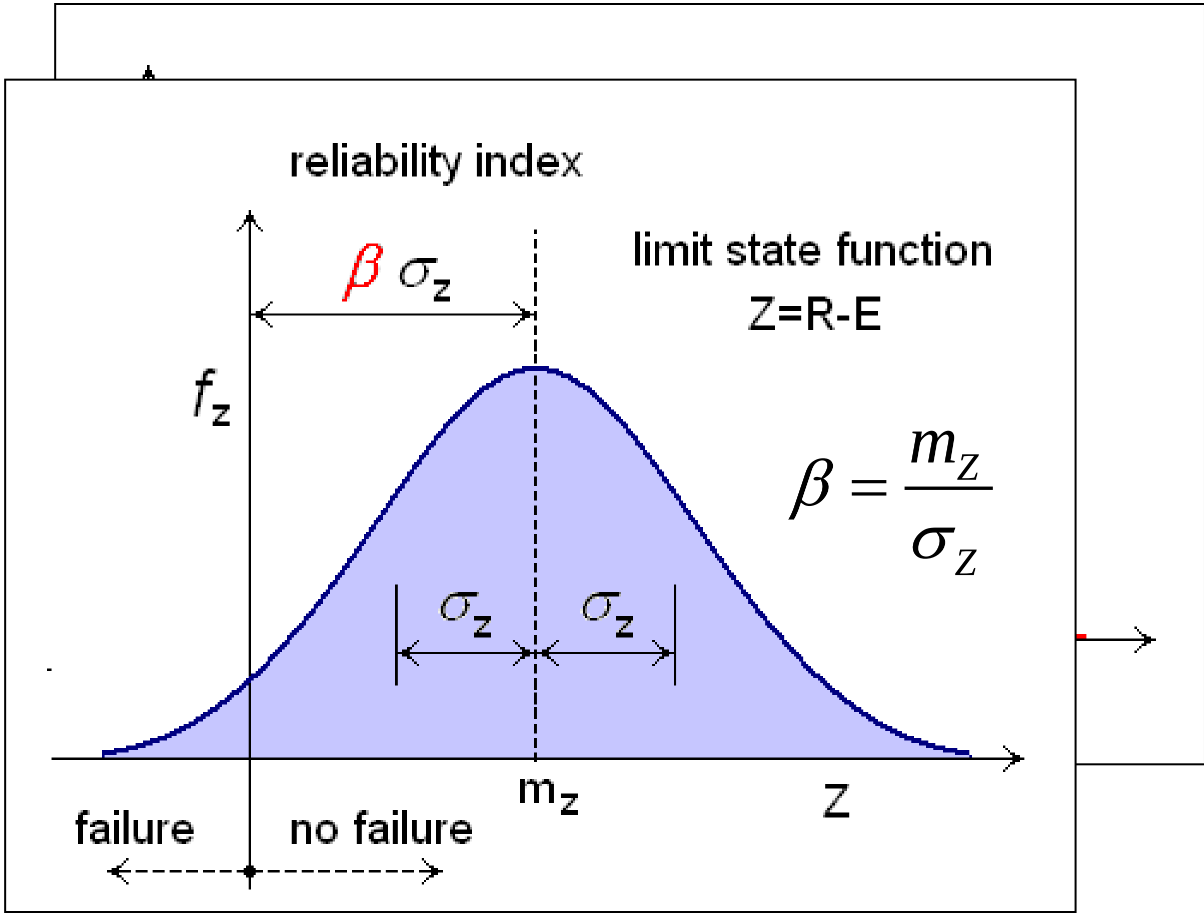
Safety and Reliability Factors

Safety factors

scatter – not considered

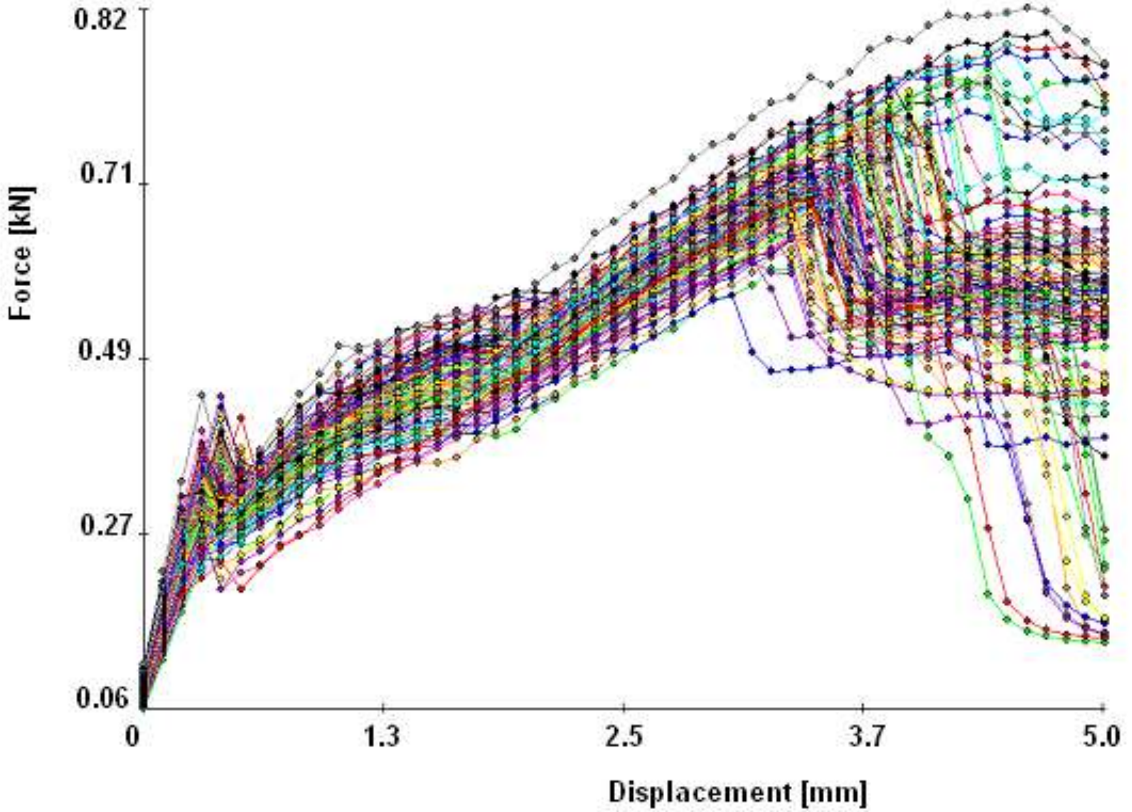
Probabilistic approach

scatter - considered

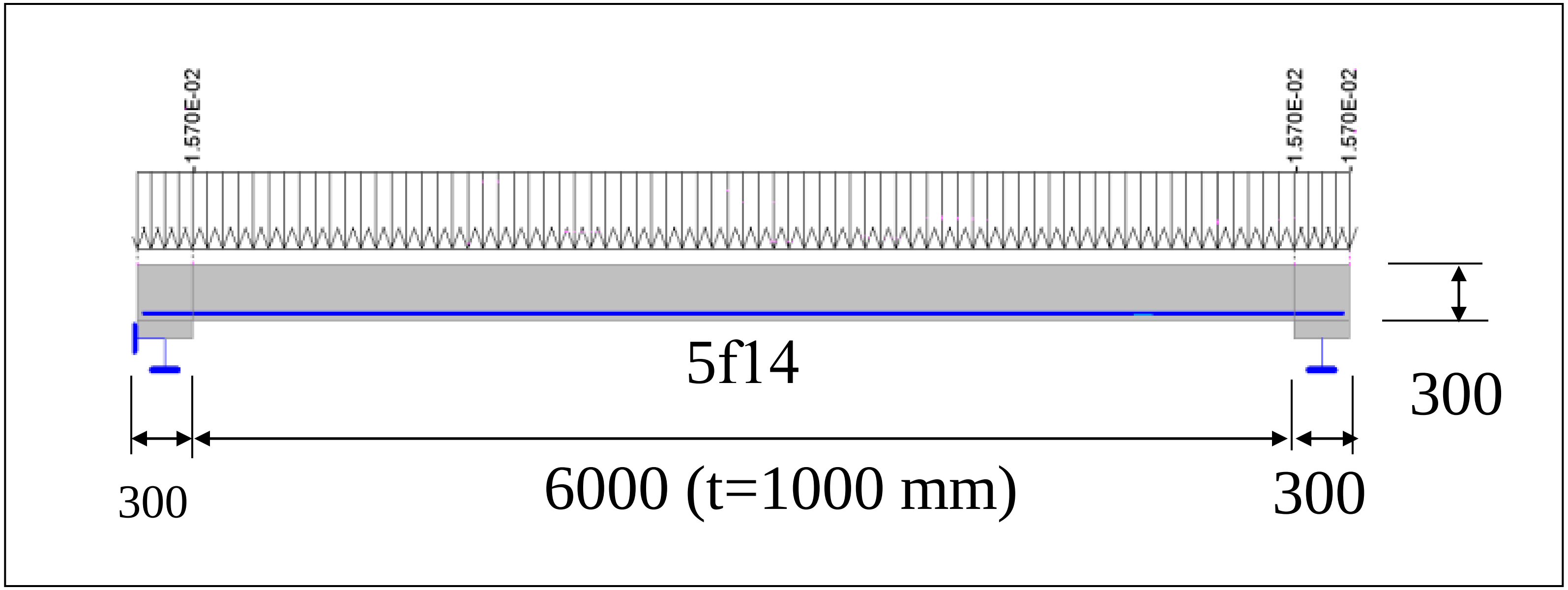


Results – probabilistic, SARA+ATENA

120 samples



Bending Beam



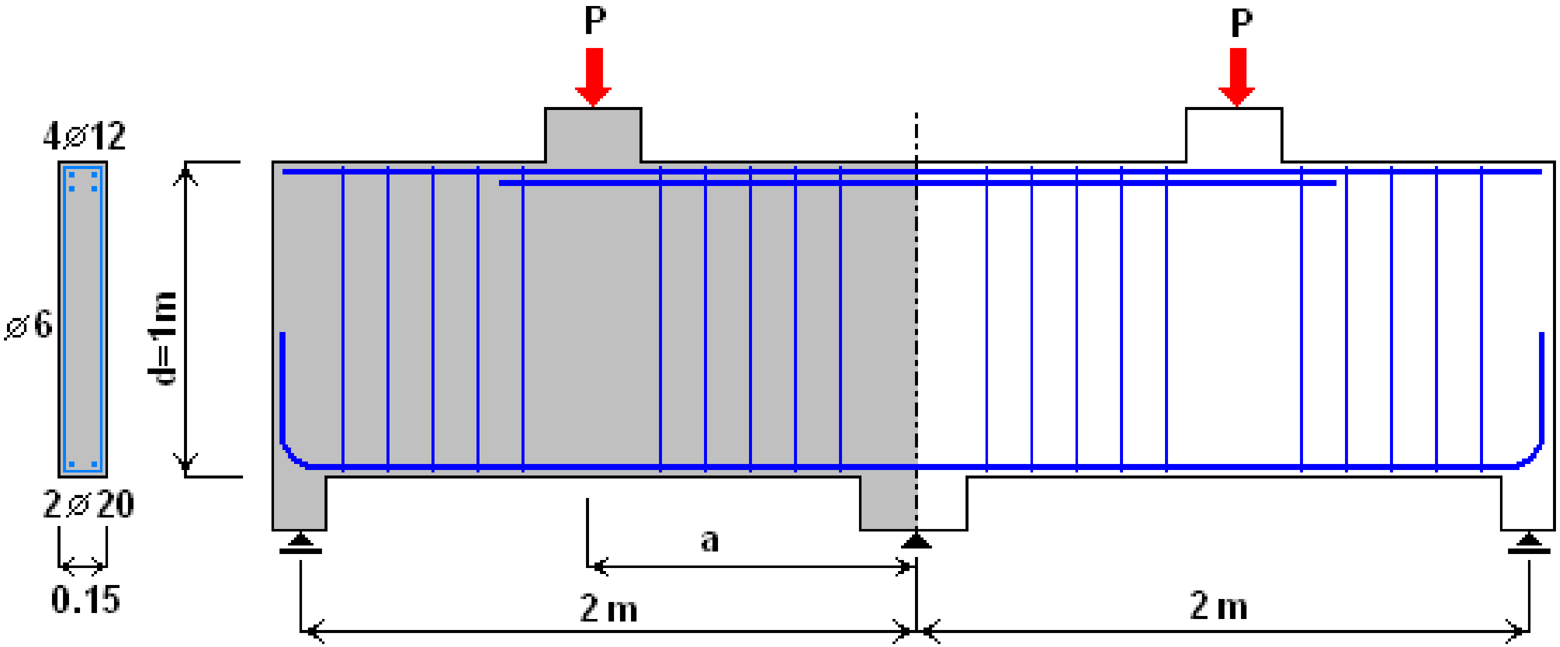
$$M_{Ed} = 77.9\,kNm < \boxed{M_{Rd} = 93\,kNm}$$

Example: Deep beam

Tested by: **Melvin Asin, Delft University, 1999**

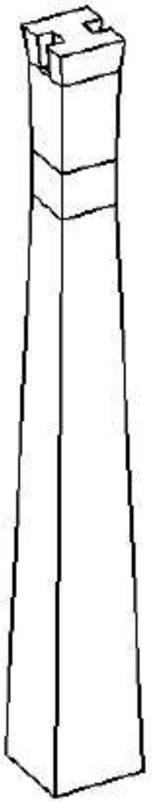
Nonlinear, probabilistic analysis by: **ATENA, 2006**

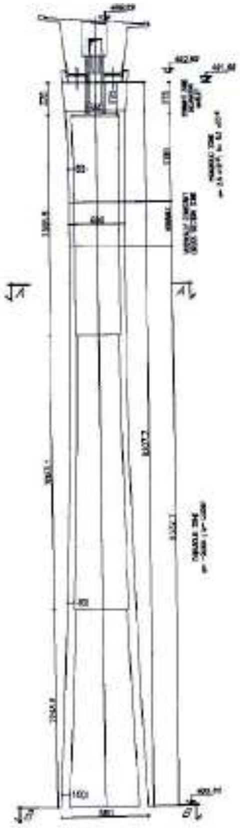
Deep beam

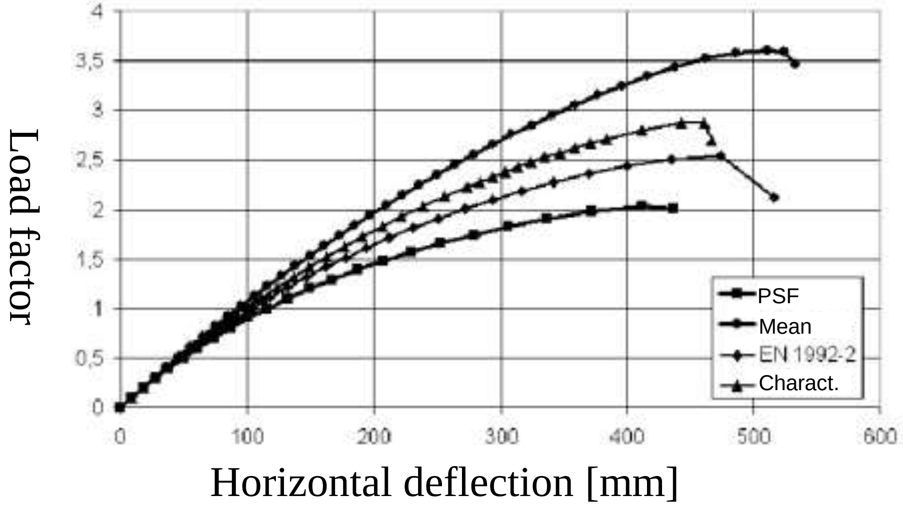


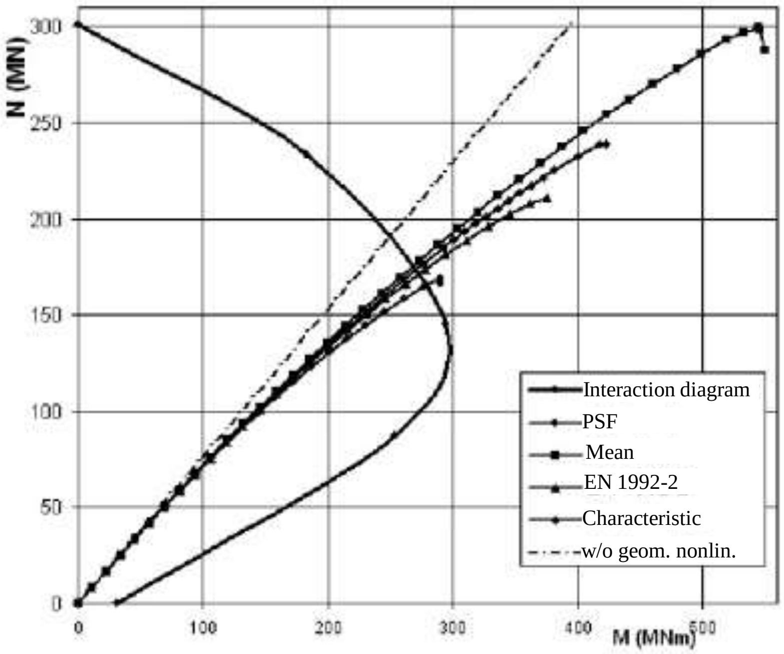
Asin: BM1/2/3 a/d=1 C30/37 S500

Reinforced Concrete Bridge Pier

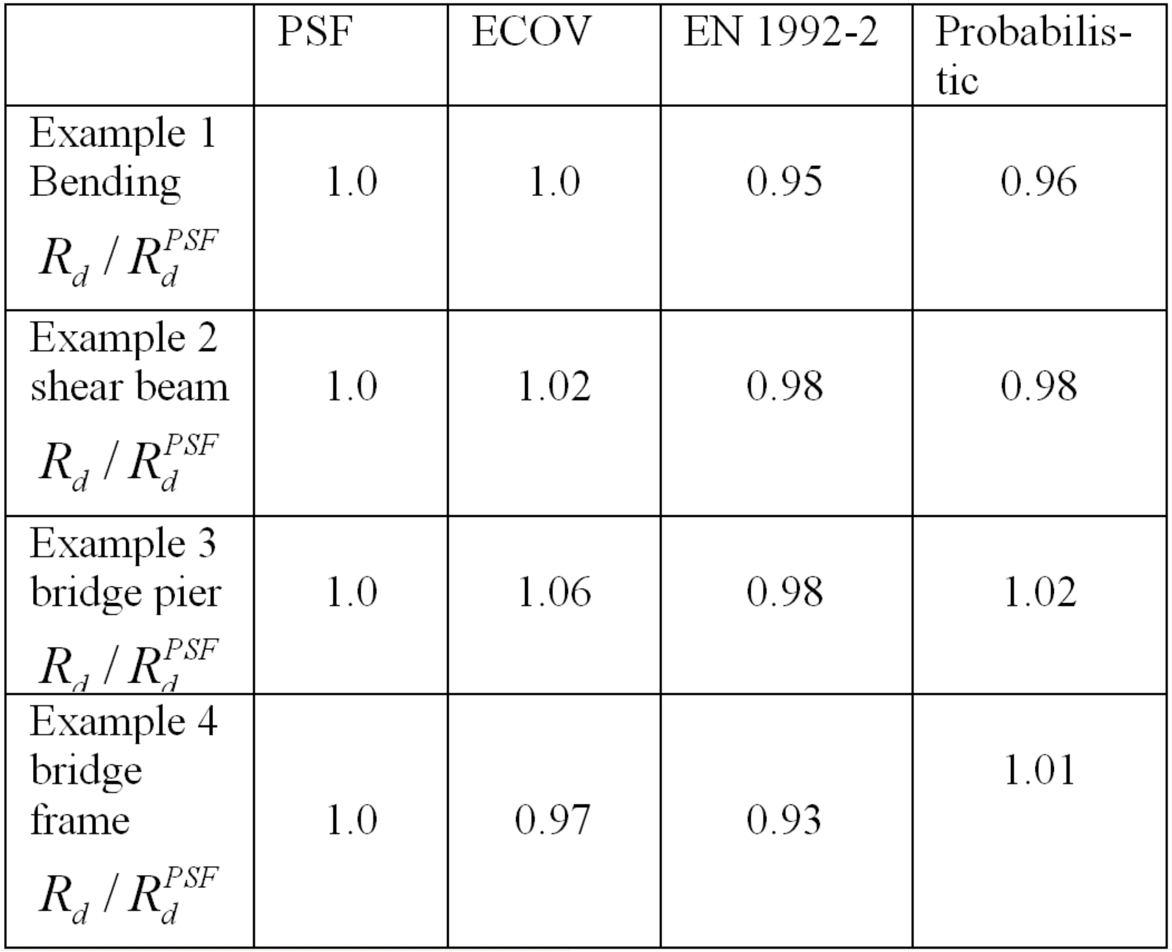






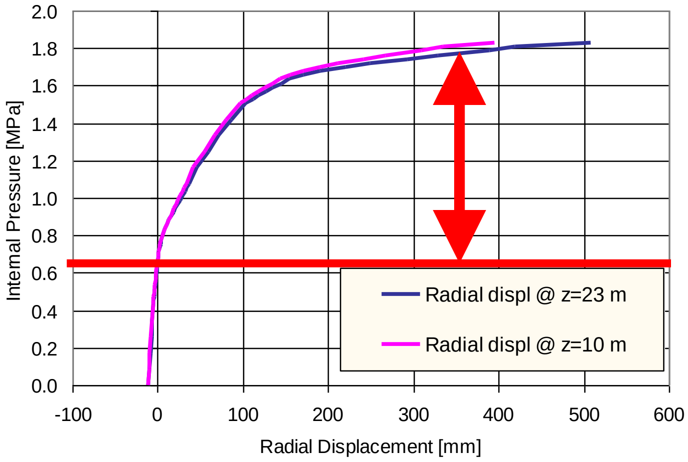


Safety Formats Comparison



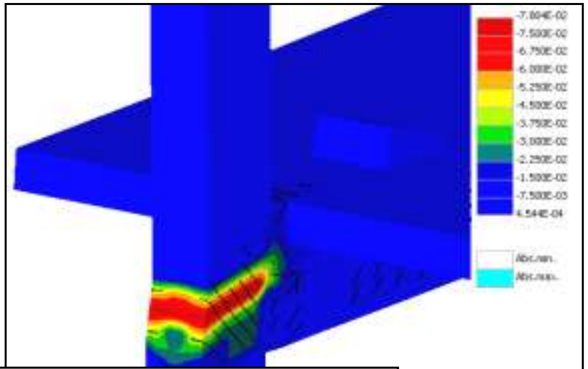
	PSF	ECOV	EN 1992-2	Probabilis-tic
Example 1 Bending R_d / R_d^{PSF}	1.0	1.0	0.95	0.96
Example 2 shear beam R_d / R_d^{PSF}	1.0	1.02	0.98	0.98
Example 3 bridge pier R_d / R_d^{PSF}	1.0	1.06	0.98	1.02
Example 4 bridge frame R_d / R_d^{PSF}	1.0	0.97	0.93	1.01

Types of Nonlinear Analysis

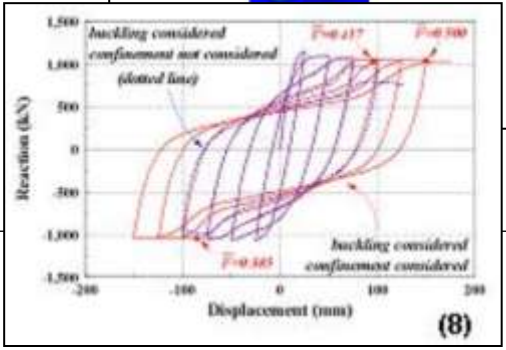


Ultimate Limit State – max load (ULS)

Service Limit State – deflection (SLS) crack width



Seismic Assessment (SA) pushover accelerogram



Types of Nonlinear Analysis

Structural details

reinforcement detailing

special details

problems with
boundary conditions

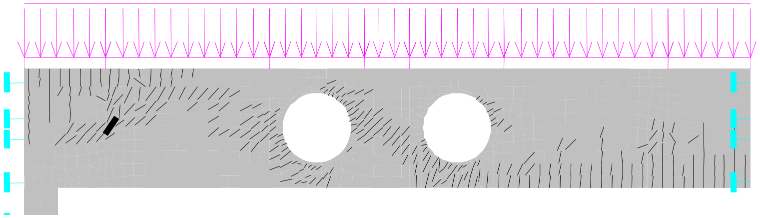
Overall structural behaviour

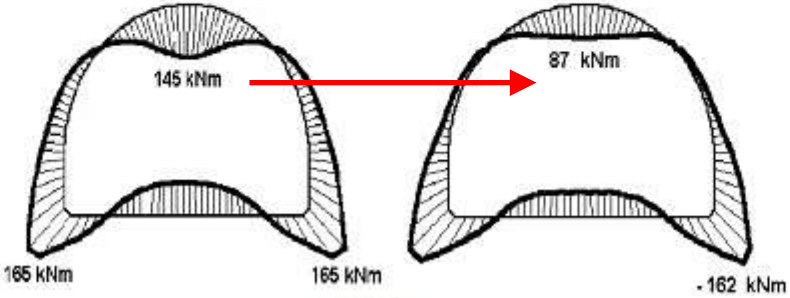
redistribution due to cracking

ULS, SLS, SA

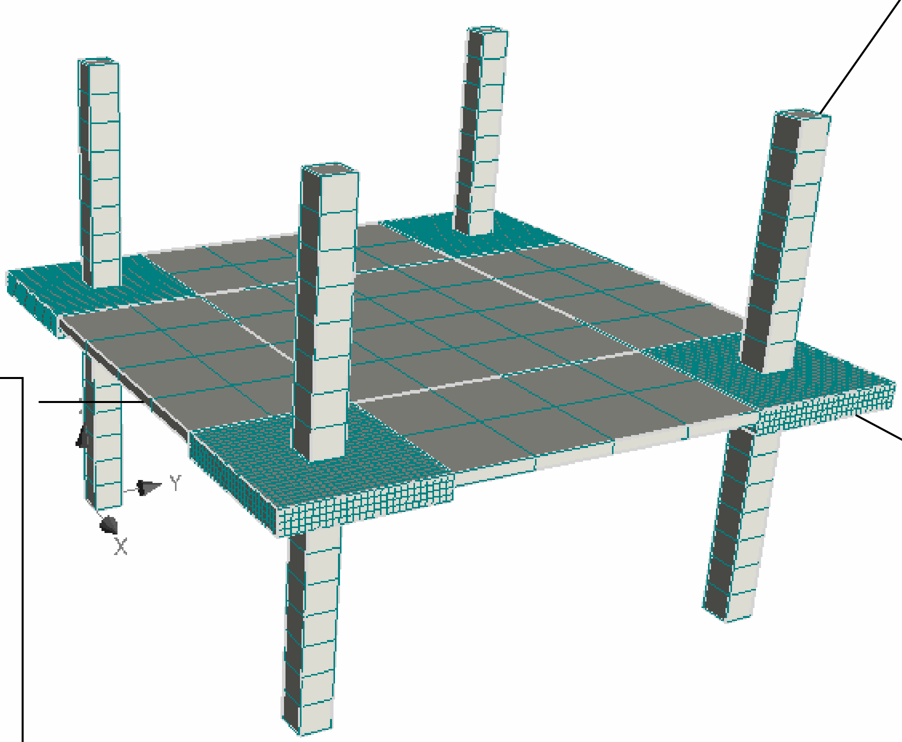
bending OK

shear or other local effects
not modelled??





Modelling Issues for nonlinear analyses of RC



Columns bending failure expected

Use beam elements with fibres

Bending failure expected

Use shell elements

Shear + bending failure

Use solid elements

Conclusions

Simulation by nonlinear analysis is used as a standard tool in design practice or for the evaluation of existing structures

- Removes inconsistency in standard design process between linear analysis and non-linear cross-section check
- Provides insight into the structural behavior
- Helps to discover critical locations and failure modes
- May discover additional load-carrying capacity
- Ideal tool for checking reinforcement detailing in complicated D-regions

State of art:

- “Complexity” -> old myth from the 20th century
- Available in many commercial finite element codes
- Computationally more demanding than linear analysis
- Supplement standard design based on linear analysis and section design

Thank you for your attention